9-4 Models for Population Growth

2.

(a) $\frac{dP}{dt} = 0.0015P(1 - \frac{P}{6000})$

(e) $A = \frac{6000 - 1000}{1000} = 5$, $P(t) = \frac{6000}{1 + 5e^{-0.0015t}}$

4.

(a) carrying capacity:684

(b) $k = \frac{1}{18}\frac{39 - 18}{2 - 0} = \frac{1}{18}\frac{21}{2} = \frac{7}{12}$

(c) exponential model: $P(t) = 18e^{\frac{7}{12}t}$, logisitc model:$P(t) = \frac{684}{1 + 37e^{-\frac{7}{12}t}}$, where $A = \frac{684 - 18}{18} = 37$

(e) $P(7) = \frac{684}{1 + 37e^{-\frac{7}{12}7}} \approx \frac{684}{1 + 37e^{-4.08}} \approx 403.397 \approx 403$

6.

(a) I guess the carrying capacity for the US population is 300 million. $A = \frac{300 - 250}{250} = 0.2 \Rightarrow P(t) = \frac{300}{1 + 0.2e^{-kt}}$

(b) $P(10) = 275 \Rightarrow 275 = \frac{300}{1 + 0.2e^{-10k}} \Rightarrow 275 + 55e^{-10k} = 300 \Rightarrow 55e^{-10k} = 25 \Rightarrow 11e^{-10k} = 5 \Rightarrow \ln 11 - 10k = \ln 5 \Rightarrow k = \frac{1}{10}\ln\frac{11}{5}$

(c) $P(110) = \frac{300}{1 + 0.2e^{-\frac{1}{10}\ln\frac{11}{5}110}} = \frac{300}{1 + 0.2(\frac{5}{11})^{11}}$, $P(210) = \frac{300}{1 + 0.2e^{-\frac{1}{10}\ln\frac{11}{5}210}} = \frac{300}{1 + 0.2(\frac{5}{11})^{21}}$

8.

(a) $A = \frac{10000 - 400}{400} = 24$, $P(t) = \frac{10000}{1 + 24e^{-kt}}$, $P(1) = 1200 \Rightarrow 1200 = \frac{10000}{1 + 24e^{-k}} \Rightarrow k = \ln\frac{36}{11} \Rightarrow P(t) = \frac{10000}{1 + 24e^{-\ln\frac{36}{11}t}}$

(b) $5000 = \frac{10000}{1 + 24e^{-(\ln\frac{36}{11})t}} \Rightarrow t = \frac{\ln 24}{\ln\frac{36}{11}} = \frac{\ln 24}{\ln 36 - \ln 11}$

10. Omitted.

12. Omitted.

14.

(a) $\frac{dy}{dt} = ky^{1+c} \Rightarrow y^{-1-c}dy = kdt \Rightarrow \frac{1}{-c}y^{-c} = kt + a,\ y(0) = y_0 \Rightarrow a = \frac{1}{-cy_0^c} \Rightarrow y^{-c} = -ckt + \frac{1}{y_0^c} = \frac{-ckty_0^c + 1}{y + 0^t} \Rightarrow y^c = \frac{y_0^c}{1 - ckty_0^c} \Rightarrow y = \frac{y_0}{(1 - ckty_0^c)^{\frac{1}{c}}}$

(b) $1 - ckTy_0^c = 0 \Rightarrow cky_0^c T = 1 \Rightarrow T = \frac{1}{cky_0^c}$

(c) $c = 0.01, y_0 = 2, y(3) = 16 \Rightarrow 16 = \frac{2}{(1 - 0.03k2^{0.01})^{\frac{1}{0.01}}}$

16. Omitted.

18.

(a) $\frac{dP}{dt} = cP\ln\frac{K}{P} \Rightarrow \frac{1}{P(\ln K - \ln P)}dP = cdt \Rightarrow -\ln(\ln K - \ln P) = ct + a \Rightarrow \ln(\ln\frac{K}{P}) = -ct + a \Rightarrow \ln(\frac{K}{P}) = Ae^{-ct}, A = e^a \Rightarrow \frac{K}{P} = e^{Ae^{-ct}} \Rightarrow P(t) = Ke^{-Ae^{-ct}}$

(b) $\lim_{t\to\infty} P(t) = K$

(c) Omitted.

(d) $\frac{d^2P}{dt^2} = c\frac{dP}{dt}\ln(\frac{K}{P}) - cP\frac{P}{K}KP^{-2}\frac{dP}{dt} = c\frac{dP}{dt}\ln\frac{K}{P} - c\frac{dP}{dt} = c[\frac{dP}{dt}\ln(\frac{K}{P}) - \frac{dP}{dt}] = 0 \Rightarrow \frac{dP}{dt}[\ln(\frac{K}{P}) - 1] = 0 \Rightarrow c\ln(\frac{K}{P})P[\ln(\frac{K}{P}) - 1] = 0 \Rightarrow P = K or P = 0 or \ln\frac{K}{P} = 1 \Rightarrow P = \frac{K}{e}$

20.

(a) $\frac{dP}{dt} = kP\cos^2(rt - \phi) \Rightarrow \frac{1}{P}dp = k\cos^2(rt - \phi)dt \Rightarrow \ln P = k\int\frac{1 + \cos(2rt - 2\phi)}{2}dt = \frac{k}{2}t + \frac{k}{2}\sin(2rt - 2\phi)\frac{1}{2r} = \frac{k}{2}t + \frac{k}{4r}\sin(2rt - 2\phi)$