9.3

30 $y^2 = kx^3 \Rightarrow 2y\frac{dy}{dx} = 3kx^2 \Rightarrow \frac{dy}{dx} = \frac{3kx^2}{2y}$

the orthogonal family satisfies $\frac{dy}{dx} = \frac{-2y}{3kx^2}$

$\Rightarrow \frac{-1}{2y}dy = 3kx^2dx \Rightarrow \int \frac{-1}{2y}dy = \int 3kx^2dx \Rightarrow \frac{-1}{2}ln\,|y| = kx^3 + c$

32 $y = \frac{x}{1+kx}\;\frac{dy}{dx} = \frac{1+kx-kx}{(1+kx)^2} = \frac{1}{(1+kx)^2} \Rightarrow\; dy = \frac{1}{(1+kx)^2}dx \Rightarrow \frac{dy}{dx} = \frac{1}{(1+kx)^2}$

the orthogonal family satisfies $\frac{dy}{dx} = -(1+kx)^2$

$\Rightarrow dy = -(1+kx)^2dx \Rightarrow \int dy = \int -(1+kx)^2dx$

$y = \frac{-(1+kx)^3}{3k} + c$

34 let the temperature of the coffee be x(t)

$\frac{dx}{dt} = -C(x-20) \Rightarrow 1 = -C(70-20) \Rightarrow C = -0.02$

$\frac{1}{x-20}dx = -0.02dt \Rightarrow \int \frac{1}{x-20}dx = -0.02t + d$

$ln(x-20) = -0.02t + d \Rightarrow x - 20 = e^{-0.02t+d}$

$\Rightarrow x(t) = 20 + e^{-0.02t+d} \Rightarrow x(0) = 20 + e^d = 95$

$\Rightarrow d = ln75 \Rightarrow x(t) = 20 + 75e^{-0.02t}$

36 suppose the concentration of C is x(t),

then the concentration of A is a-x(t) and B is b-x(t)

$\frac{dx}{dt} = \frac{d[C]}{dt} = k[A][B] = k(x-a)(x-b)$

38 $\frac{d^2T}{dr^2} + \frac{2dT}{rdr} = 0\; S = \frac{dT}{dr} \Rightarrow \frac{dS}{dr} = \frac{2S}{r}$

$\Rightarrow \frac{dS}{2S} = \frac{1}{r}dr \Rightarrow \frac{1}{2}lnS = lnr + c \Rightarrow lnS = 2lnr + 2c$

$\Rightarrow S = r^2e^{2c} \Rightarrow \frac{dT}{dr} = r^2e^{2c} \Rightarrow T(r) = \frac{1}{3}r^3e^{2c} + d$

$T(1) = 15 \Rightarrow \frac{1}{3}e^{2c} + d = 15\; T(2) = 25 \Rightarrow \frac{4}{3}e^{2c} + d = 25$

$\Rightarrow c = \frac{1}{2}ln10\; d = \frac{35}{3} \Rightarrow T(r) = \frac{10}{3}r^3 + \frac{35}{3}$

40(a)let the amount of new currency be x(t)

$\frac{dx}{dt} = 50,000 \times (1 - \frac{x}{10,000,000}) \Rightarrow \frac{dx}{dt} = 50,000 - 0.005x$

$(b)\frac{1}{50,000-0.005x}dx = dt \Rightarrow \int \frac{1}{50,000-0.005x}dx = t +$

$\Rightarrow -200ln\,|50,000 - 0.005x| = t + c \Rightarrow 50,000 - 0.005x = e^{-0.005t+c}$

$x(0) = 0 \Rightarrow e^c = 50,000 \Rightarrow x(t) = 10,000,000(1 - e^{-0.005t})$

$(c)\frac{x(t)}{10,000,000} = 0.9$

$\Rightarrow 1 - e^{-0.005t} = 0.9 \Rightarrow e^{-0.005t} = 0.1 \Rightarrow t = ln(0.1) \times (-200) = 460.5(day)$

42 let the amount of carbon dioxide be x(t)

$\frac{dx}{dt} = 2 \times 0.0005 - \frac{2}{180}x \Rightarrow \frac{dx}{dt} = 0.001 - \frac{1}{90}x \Rightarrow 90dx = (0.09 - x)dt$

$\Rightarrow \frac{1}{0.09-x}dx = \frac{1}{90}dt \Rightarrow -ln\,|0.09 - x| = \frac{1}{90}t + c$

when t=0 x=180x0.0015=0.27$\Rightarrow -ln\,|-0.18| = c$

$\Rightarrow \frac{1}{|0.09-x|} = \frac{50}{9}e^{\frac{t}{90}} \Rightarrow |0.09 - x| = \frac{9}{50}e^{\frac{-t}{90}}$

$\Rightarrow lim_{t\longrightarrow\infty}\,|0.09 - x| = 0 \Rightarrow lim_{t\longrightarrow\infty}x = 0.09$

the concentrate of carbon dioxide becomes 0.05 percent at last

44 let the amount of salt be x(t)

$\frac{dx}{dt} = 5 \times 0.05 + 10 \times 0.04\frac{dx}{dt} = 0.25 + 0.4 = 0.29$

$\Rightarrow x(t) = 0.29t + c \Rightarrow x(0) = 0 \Rightarrow c = 0$

$(a)0.29t$kg salt in the tank$(b)0.29 \times 60$kg salt in the tank

$46(a) m\frac{dv}{dt} = -kv \Rightarrow \frac{1}{v}dv = \frac{-k}{m}dt$
$\Rightarrow lnv = \frac{-kt}{m} + c \Rightarrow v = e^{\frac{-kt}{m}+c} = v_0 e^{\frac{-kt}{m}}$
$\frac{ds}{dt} = v_0 e^{\frac{-kt}{m}} \Rightarrow s = \frac{-m}{k} v_0 e^{\frac{-kt}{m}} + d$
$\Rightarrow s(0) = \frac{-m}{k} v_0 + d = s_0 \Rightarrow d = s_0 + \frac{m}{k} v_0$
$s(t) = \frac{m}{k} v_0 (1 - e^{\frac{-kt}{m}}) + s_0$

$(b) m\frac{dv}{dt} = -kv^2 \Rightarrow \frac{1}{v^2}dv = \frac{-k}{m}dt$
$\Rightarrow \int \frac{1}{v^2}dv = \int \frac{-k}{m}dt \Rightarrow -\frac{1}{v} = \frac{-kt}{m} + c$
$\Rightarrow v = \frac{m}{kt-cm} \Rightarrow v(0) = \frac{-1}{c} = v_0$
$\Rightarrow c = \frac{-1}{v_0} \Rightarrow v = \frac{mv_0}{v_0 kt+m}$
$\frac{ds}{dt} = \frac{mv_0}{v_0 kt+m} \Rightarrow s = \int \frac{mv_0}{v_0 kt+m}dt = \frac{m}{k} ln\left|v_0 kt + m\right| + d$
$\Rightarrow s(0) = \frac{m}{k} lnm + d = s_0 \Rightarrow d = s_0 - \frac{m}{k} lnm$

$48(a) m\frac{dv}{dt} = -\frac{mgR^2}{(x+R)^2}$
$m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx} \Rightarrow mv\frac{dv}{dx} = -\frac{mgR^2}{(x+R)^2}$
$\Rightarrow mvdv = -\frac{mgR^2}{(x+R)^2}dx \int mvdv = \int -\frac{mgR^2}{(x+R)^2}dx$
$\frac{mv^2}{2} = \frac{mgR^2}{x+R} + c$
when v=0 x reaches the maximum hieght$\Rightarrow c = -\frac{mgR^2}{h+R}$
$\Rightarrow mv_0^2 = 2(mgR - \frac{mgR^2}{h+R}) = \frac{2mgRh}{h+R} \Rightarrow v_0 = \sqrt{\frac{2gRh}{h+R}}$

$(b) v_e = lim_{h\longrightarrow\infty} v_0 = lim_{h\longrightarrow\infty}\sqrt{\frac{2gRh}{h+R}} = \sqrt{2gR}$

$(c) v_e = \sqrt{2 \times 9.8 \times 6370000} \approx 11173.7(m/s)$
$v_e = 11173.7 \times 60 \div 1000 = 670.4(km/min)$