# 9.2 DIRECTION FIELDS AND EULER'S METHOD

10. The level curves of $y'$ are hyperbolas

$$x^2 - y^2 = m$$

12. The level curves of $y'$ are hyperbolas

$$xy = 1 - m$$

14. The level curves of $y'$ are hyperbolas

$$x(y-1) = -m$$

18. The level curves of $y'$ are horizontal lines, and the slope field are symmetric about $x-$axis. When $y(0) < -1$ , $y \to -2$ as $x \to \infty$. When $y(0) = -1$ , $y \equiv 1$. When $-1 < y(0) < 2$ , $y \to 1$ as $x \to \infty$. When $y(0) = 2$ , $y \equiv 2$. When $y(0) > 2$ , $y \to \infty$ as $x \to \infty$.

20. The level curves of $y'$ are vertical lines, and it decreases as $x$ goes larger, so Euler's method will overestimate.

22.

$$y(1) = \int_0^1 (1-xy)dx \simeq \sum_{k=0}^{n-1}(1 - \frac{k-1}{n}y_{k-1})\frac{1}{n}$$

$$y(0.2) \simeq 0.2$$
$$y(0.4) \simeq 0.2 + (1 - 0.2 * 0.2)0.2 = 0.392$$
$$y(0.6) \simeq 0.392 + (1 - 0.4 * 0.392)0.2 = 0.561$$
$$y(0.8) \simeq 0.561 + (1 - 0.6 * 0.561)0.2 = 0.694$$
$$y(1.0) \simeq 0.694 + (1 - 0.8 * 0.694)0.2 = 0.783$$

24.

$$y' = x(1-y)$$

(1)

$$y(1) = 0$$
$$y(1.2) \simeq 0.2$$
$$y(1.4) \simeq 0.2 + 1.2 * 0.8 * 0.2 = 0.392$$

(2)

$$y(1) = 0$$
$$y(1.1) \simeq 0.1$$
$$y(1.2) \simeq 0.1 + 1.1 * 0.9 * 0.1 = 0.199$$
$$y(1.3) \simeq 0.199 + 1.2 * 0.801 * 0.1 = 0.295$$
$$y(1.4) \simeq 0.295 + 1.3 * 0.705 * 0.1 = 0.387$$

26.

$$y(0) = 1$$
$$y(0.01(k+1)) = y(0.01k) + 0.01[(0.01k)^3 - (y(0.01k))^3], k = 0, 1, ..., 199$$

28. (a)

$$\frac{dT}{dt} = -k(T - T_{env}) = -k(T - 20)$$
$$1 = k * 50, k = \frac{1}{50}$$
$$y(1.4) \simeq 0.2 + 1.2 * 0.8 * 0.2 = 0.392$$

(b) The level curves of $\frac{dT}{dt}$ are horizontal lines. Moreover, T will go to 20 eventually.

(c)

$$T(0) = 95$$
$$T(2) \simeq 95 - \frac{1}{50} * 75 * 2 = 92$$
$$T(4) \simeq 92 - \frac{1}{50} * 72 * 2 = 89.12$$
$$T(6) \simeq 89.12 - \frac{1}{50} * 69.12 * 2 = 85.36$$
$$T(8) \simeq 85.36 - \frac{1}{50} * 65.36 * 2 = 82.75$$
$$T(10) \simeq 82.75 - \frac{1}{50} * 62.75 * 2 = 80.24 < T(10)$$

In fact, $T(10) = 20 + 75e^{-0.2}$