Solutions of 9.1

2. $y' = \cos^2 x - \sin^2 x + \sin x$. Direct calculation shows that y is a solution of the initial-value problem.

4. (a) $y = \cos kt \Rightarrow y'' = -k^2 \cos kt$. For y to satisfy the equation $y'' = -25y$, we need $k = \pm 5$.

   (b) The verification is straightforward.

6. (a) $y = Ce^{x^2/2} \Rightarrow y' = Ce^{x^2/2} \cdot x = xy$.

   (c) Let y be as in (a), then $y(0) = C$. So the required solution is $y = 5e^{x^2/2}$.

   (d) Let y be as in (a), then $y(1) = Ce^{1/2}$. So $y(1) = 2 \Rightarrow C = 2e^{-1/2}$. And so the solution is $y = 2e^{(x^2-1)/2}$.

8. (a) If x is close to 0, then xy^3 is close to 0, and hence y' is close to 0. If x is large, then xy^3 is large and so y' is large.

   (b) The verification is straightforward.

   (d) $y = (c - x^2)^{-1/2} \Rightarrow y(0) = c^{-1/2}$. So $y(0) = 2 \Rightarrow c = \frac{1}{4}$, and so $y = (\frac{1}{4} - x^2)^{-1/2}$.

10. (a) If y is a constant function, then $\frac{dy}{dt} = 0$, and hence $y^4 - 6y^3 + 5y^2 = 0$. So $y = 0$, 1, or 5.

    (b) y increasing $\Leftrightarrow \frac{dy}{dt} = y^2(y-1)(y-5) \geq 0 \Leftrightarrow y \geq 5$ or $y \leq 1$.

    (c) y decreasing $\Leftrightarrow 1 \leq y \leq 5$.

12. Note that if $x = 0$, we have $y' > 0$; and for positive y, if x is large enough we should have $y' < 0$. The only possibility is **C.**

14. (a) The coffee cools most quickly as soon as it is removed from the heat source. The rate of cooling decreases toward 0 since the coffee approaches room temperature.

    (b) $\frac{dy}{dt} = k(y - R)$, where k is a proportionality constant, y is the temperature of the coffee, and R is the room temperature. The initial condition is $y(0) =$ 95°C. The answer in (a) and the model support each other because as y approaches R, $\frac{dy}{dt}$ approaches 0, so the model seems appropriate.