10. Let $u = \sqrt{2}\,y$ and $a = \sqrt{3}$. Then $du = \sqrt{2}\,dy$ and

$$\int \frac{\sqrt{2y^2-3}}{y^2}\,dy = \int \frac{\sqrt{u^2-a^2}}{\frac{1}{2}u^2}\,\frac{du}{\sqrt{2}} = \sqrt{2}\int \frac{\sqrt{u^2-a^2}}{u^2}\,du \overset{42}{=} \sqrt{2}\left(-\frac{\sqrt{u^2-a^2}}{u} + \ln\left|u+\sqrt{u^2-a^2}\right|\right) + C$$

$$= \sqrt{2}\left(-\frac{\sqrt{2y^2-3}}{\sqrt{2}\,y} + \ln\left|\sqrt{2}\,y+\sqrt{2y^2-3}\right|\right) + C$$

$$= -\frac{\sqrt{2y^2-3}}{y} + \sqrt{2}\ln\left|\sqrt{2}\,y+\sqrt{2y^2-3}\right| + C$$

14. Let $u = \sqrt{x}$. Then $u^2 = x$ and $2u\,du = dx$, so

$$\int \sin^{-1}\sqrt{x}\,dx = 2\int u\sin^{-1}u\,du \overset{90}{=} \frac{2u^2-1}{2}\sin^{-1}u + \frac{u\sqrt{1-u^2}}{2} + C = \frac{2x-1}{2}\sin^{-1}\sqrt{x} + \frac{\sqrt{x(1-x)}}{2} + C.$$

17. Let $z = 6+4y-4y^2 = 6-(4y^2-4y+1)+1 = 7-(2y-1)^2$, $u = 2y-1$, and $a = \sqrt{7}$. Then $z = a^2-u^2$, $du = 2\,dy$, and

$$\int y\sqrt{6+4y-4y^2}\,dy = \int y\sqrt{z}\,dy = \int \tfrac{1}{2}(u+1)\sqrt{a^2-u^2}\,\tfrac{1}{2}\,du = \tfrac{1}{4}\int u\sqrt{a^2-u^2}\,du + \tfrac{1}{4}\int\sqrt{a^2-u^2}\,du$$

$$= \tfrac{1}{4}\int\sqrt{a^2-u^2}\,du - \tfrac{1}{8}\int(-2u)\sqrt{a^2-u^2}\,du$$

$$\overset{30}{=} \frac{u}{8}\sqrt{a^2-u^2} + \frac{a^2}{8}\sin^{-1}\left(\frac{u}{a}\right) - \frac{1}{8}\int\sqrt{w}\,dw \qquad \begin{bmatrix} w = a^2-u^2, \\ dw = -2u\,du \end{bmatrix}$$

$$= \frac{2y-1}{8}\sqrt{6+4y-4y^2} + \frac{7}{8}\sin^{-1}\frac{2y-1}{\sqrt{7}} - \frac{1}{8}\cdot\frac{2}{3}w^{3/2} + C$$

$$= \frac{2y-1}{8}\sqrt{6+4y-4y^2} + \frac{7}{8}\sin^{-1}\frac{2y-1}{\sqrt{7}} - \frac{1}{12}(6+4y-4y^2)^{3/2} + C$$

This can be rewritten as

$$\sqrt{6+4y-4y^2}\left[\frac{1}{8}(2y-1) - \frac{1}{12}(6+4y-4y^2)\right] + \frac{7}{8}\sin^{-1}\frac{2y-1}{\sqrt{7}} + C$$

$$= \left(\frac{1}{3}y^2 - \frac{1}{12}y - \frac{5}{8}\right)\sqrt{6+4y-4y^2} + \frac{7}{8}\sin^{-1}\left(\frac{2y-1}{\sqrt{7}}\right) + C$$

$$= \frac{1}{24}(8y^2-2y-15)\sqrt{6+4y-4y^2} + \frac{7}{8}\sin^{-1}\left(\frac{2y-1}{\sqrt{7}}\right) + C$$

19. Let $u = \sin x$. Then $du = \cos x\,dx$, so

$$\int \sin^2 x\,\cos x\,\ln(\sin x)\,dx = \int u^2 \ln u\,du \overset{101}{=} \frac{u^{2+1}}{(2+1)^2}\left[(2+1)\ln u - 1\right] + C = \tfrac{1}{9}u^3(3\ln u - 1) + C$$
$$= \tfrac{1}{9}\sin^3 x\,[3\ln(\sin x) - 1] + C$$

20. Let $u = \sin\theta$, so that $du = \cos\theta\,d\theta$. Then

$$\int \frac{\sin 2\theta}{\sqrt{5 - \sin\theta}}\,d\theta = \int \frac{2\sin\theta\,\cos\theta}{\sqrt{5 - \sin\theta}}\,d\theta = 2\int \frac{u}{\sqrt{5 - u}}\,du \overset{55}{=} 2 \cdot \frac{2}{3(-1)^2}\left[-1u - 2(5)\right]\sqrt{5 - u} + C$$
$$= \tfrac{4}{3}(-u - 10)\sqrt{5 - u} + C = -\tfrac{4}{3}(\sin\theta + 10)\sqrt{5 - \sin\theta} + C$$

33. (a) $$\frac{d}{du}\left[\frac{1}{b^3}\left(a + bu - \frac{a^2}{a + bu} - 2a\ln|a + bu|\right) + C\right] = \frac{1}{b^3}\left[b + \frac{ba^2}{(a + bu)^2} - \frac{2ab}{(a + bu)}\right]$$
$$= \frac{1}{b^3}\left[\frac{b(a + bu)^2 + ba^2 - (a + bu)2ab}{(a + bu)^2}\right] = \frac{1}{b^3}\left[\frac{b^3u^2}{(a + bu)^2}\right] = \frac{u^2}{(a + bu)^2}$$

(b) Let $t = a + bu \quad\Rightarrow\quad dt = b\,du$. Note that $u = \dfrac{t - a}{b}$ and $du = \dfrac{1}{b}\,dt$.

$$\int \frac{u^2\,du}{(a + bu)^2} = \frac{1}{b^3}\int \frac{(t - a)^2}{t^2}\,dt = \frac{1}{b^3}\int \frac{t^2 - 2at + a^2}{t^2}\,dt = \frac{1}{b^3}\int \left(1 - \frac{2a}{t} + \frac{a^2}{t^2}\right)dt$$
$$= \frac{1}{b^3}\left(t - 2a\ln|t| - \frac{a^2}{t}\right) + C = \frac{1}{b^3}\left(a + bu - \frac{a^2}{a + bu} - 2a\ln|a + bu|\right) + C$$

44. None of Maple, Mathematica and Derive is able to evaluate $\int (1 + \ln x)\sqrt{1 + (x\ln x)^2}\,dx$. However, if we let $u = x\ln x$, then $du = (1 + \ln x)\,dx$ and the integral is simply $\int \sqrt{1 + u^2}\,du$, which any CAS can evaluate. The antiderivative is $\frac{1}{2}\ln\left(x\ln x + \sqrt{1 + (x\ln x)^2}\right) + \frac{1}{2}x\ln x\sqrt{1 + (x\ln x)^2} + C$.