(7). $\int_{-1}^{1} \frac{e^{tan^{-1}y}}{1+y^2}dy = \int_{-1}^{1} e^{tan^{-1}y}d(tan^{-1}y) = e^{tan^{-1}y}|_{-1}^{1} = e^{\frac{\pi}{4}} - e^{-\frac{\pi}{4}} = \sqrt{2}i.$

(8). $\int x\csc x\cot xdx = \int xd-\csc x = -x\csc x+\int \csc xdx = -x\csc x+\ln|\csc x-\cot x|+C.$

(15). Let $x = \sin\theta$, $\int \frac{1}{(1-x^2)^{3/2}}dx = \int \frac{1}{\cos^3\theta}\cos\theta d\theta = \int \sec^2\theta d\theta = \tan\theta + C = \frac{x}{\sqrt{1-x^2}} + C.$

(17). $\int x\sin^2 xdx = \int x\frac{1-\cos 2x}{2}dx = \frac{1}{4}x^2-\frac{1}{2}\int x\cos 2xdx = \frac{1}{4}x^2-\frac{1}{2}[\frac{1}{2}x^2\cos 2x-\int \frac{1}{2}\sin 2xdx] = \frac{1}{4}x^2 - \frac{1}{4}x\cos 2x + \frac{1}{4}\frac{-1}{2}\cos 2x + C = \frac{1}{4}x^2 - \frac{1}{4}x\cos 2x - \frac{1}{8}\cos 2x + C.$

(23). Let $\sqrt{x} = u$, $\int_0^1(1+\sqrt{x})^8 = \int_0^1(1+u)^8 2udu = [2u\frac{1}{9}(1+u)^9]_0^1 - \int_0^1 \frac{1}{9}(1+u)^9 2du = \frac{2}{9}2^9 - [\frac{2}{9}\frac{1}{10}(1+u)^{10}]_0^1 = \frac{4097}{45}.$

(36). Let $u = \sin 4x$, $I = \int \sin 4x\cos 3xdx = \sin 4x\frac{1}{3}\sin 3x - \int \frac{1}{3}\sin 3x4\cos 4xdx = \frac{1}{3}\sin 4x\sin 3x-\frac{4}{3}[\cos 4x\frac{-1}{3}\cos 3x-\int -\frac{1}{2}\cos 3x(-4)\sin 4xdx = \frac{1}{3}\sin 4x\sin 3x+\frac{4}{9}\cos 4x\cos 3x+\frac{16}{9}I]$, hence $I = \frac{=}{9}7[\frac{1}{3}\sin 4x\sin 3x + \frac{4}{9}\cos 4x\cos 3x] = -\frac{1}{7}[3\sin 4x\sin 3x + 4\cos 4x\cos 3x].$

(38). $\int_0^{\frac{\pi}{4}}\tan^5\theta\sec^2\theta d\theta = \int_0^{\frac{\pi}{4}}(\sec^2\theta - 1)^2\sec^2\theta(\tan\theta\sec\theta d\theta) = \int_1^{\sqrt{2}}(u^2-1)^2u^2du = \int_1^{\sqrt{2}}(u^6 - 2u^4 + u^2)du = [\frac{1}{7}u^7 - \frac{2}{5} + \frac{1}{3}u^3]_1^{\sqrt{2}} = \frac{2}{105}(11\sqrt{2} - 4).$

(41). $\int \theta\tan^2\theta d\theta = \int \theta(\sec^2\theta-1)d\theta = -\frac{1}{2}\theta^2+\int \theta(\sec^2\theta d\theta) = -\frac{1}{2}\theta^2+\theta\tan\theta-\int\tan\theta d\theta = -\frac{1}{2}\theta^2 + \theta\tan\theta - \ln|\sec\theta| + C.$

(45). $\int x^5e^{-x^3}dx = \int x^3(x^2e^{-x^3}dx) = -\frac{1}{3}x^3e^{-x^3} + \frac{1}{3}\int e^{-x^3}3x^2dx = -\frac{1}{3}x^3e^{-x^3} - \frac{1}{3}e^{-x^3} + C.$

(50). Let $u = \sqrt{4x+1}$, $\int \frac{1}{x^2\sqrt{4x+1}}dx = \int \frac{16}{(u^2-1)^2u}\frac{u}{2}du = 2\int(\frac{-1}{u-1} + \frac{1}{(u-1)^2} + \frac{1}{u+1} + \frac{1}{(u+1)^2})du = 2[-\ln|u-1|\frac{1}{u-1}+\ln(u+1)-frac1u+1]+C = 2[-\ln|\sqrt{4x+1}-1|\frac{1}{\sqrt{4x+1}-1}+\ln(\sqrt{4x+1}+1) - \frac{1}{\sqrt{4x+1}+1}] + C.$

(51). Let $x = \frac{1}{2\tan\theta}$, $\int \frac{1}{x\sqrt{4x^2+1}} = \int \frac{2}{\tan\theta\sec\theta}\frac{1}{2}\sec^2\theta d\theta = \int \csc\theta d\theta = -\ln|\cot\theta+\csc\theta|+C.$

(57). Let $u = x + c$, $\int x\sqrt[3]{x+c}dx = \int(u-c)u^{1/3}du = (\frac{3}{7}u^{7/3} - \frac{3}{4}cu^{4/3}) + C = (\frac{3}{7}(x + c)^{7/3} - \frac{3}{4}c(x+c)^{4/3}) + C.$

(61). Let $u = \sqrt{x}$, $\int \sqrt{x}e^{\sqrt{x}}dx = \int ue^u 2udu = 2[u^2e^u - 2\int e^uudu] = 2[u^2e^u - 2(ue^u - inte^udu)] = 2[xe^{\sqrt{x}} - 2\sqrt{x}e^{\sqrt{x}} + 2e^{\sqrt{x}}] + C.$

(69). $\int \frac{e^{2x}}{1+e^x}dx = \int(e^x - \frac{e^x}{1+e^x})dx = e^x - \ln(1+e^x) + C.$

(78). $\int \frac{\sec x\cos 2x}{\sin x+\sec x}dx = \int \frac{\frac{1}{\cos x}(\cos^2 x-\sin^2 x)}{\sin x+\frac{1}{\cos x}}dx = \int \frac{\cos^2 x-\sin^2 x}{\sin x\cos x+1}dx = \ln(\sin x\cos x + 1) + C.$

(80). $\int \frac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx = \int \frac{\frac{1}{2}du}{u^2+(1-u)^2} = \int \frac{1}{4u^2-4u+2}du = \frac{1}{2}\int \frac{1}{1+(2u-1)^2}d(2u-1) = \frac{1}{2}\tan^{-1}(2\sin x-1)^2 + C.$