Section 7.4

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(a) $\frac{2x}{(x+3)(3x+1)} = \frac{A}{x+3} + \frac{B}{3x+1}$

(b) $\frac{1}{x^3+2x^2+x} = \frac{1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

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(a) $\frac{x^3}{x^2+4x+3} = x - 4 + \frac{13x+12}{x^2+4x+3} = x - 4 + \frac{13x+12}{(x+1)(x+3)} = x - 4 + \frac{A}{x+1} + \frac{B}{x+3}$

(b) $\frac{2x+1}{(x+1)^3(x^2+4)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{Dx+E}{x^2+4} + \frac{Fx+G}{(x^2+4)^2}$

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$\int \frac{x^2}{x+1} dx = \int (x - 1 + \frac{1}{x+1}) dx = \frac{x^2}{2} - x + \ln|x+1| + C$

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$\int \frac{1}{(t+4)(t-1)} dt = \int \frac{-1}{5}\frac{1}{(t+4)} + \frac{1}{5}\frac{1}{(t+4)} dt = \frac{-1}{5}\ln|t+4| + \frac{1}{5}\ln|t+4| + C$

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$\int_0^1 \frac{x-1}{x^2+3x+2} dx = \int_0^1 \frac{-2}{(x+1)} + \frac{3}{(x+2)} dx = [-2\ln|x+1| + 3\ln|x+2|]_0^1 =$

$3\ln 3 - 5\ln 2 = \ln\frac{27}{32}$

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$\int \frac{ds}{s^2(s-1)^2} = \int \left[\frac{2}{s} + \frac{1}{s^2} + \frac{-2}{s-1} + \frac{1}{(s-1)^2}\right] ds$

$= 2\ln|s| - \frac{1}{s} - 2\ln|s-1| - \frac{1}{s-1} + C$

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$\int \frac{10}{(x+1)(x^2+9)} dx = \int \left[\frac{1}{x-1} + (-1)\frac{x}{x^2+9} + (-1)\frac{1}{x^2+9}\right] dx$

$= \ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\tan^{-1}(\frac{x}{3}) + C$

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$\int \frac{x^3+2x^2+3x-2}{(x^2+2x+2)^2} dx = \int \left[\frac{x}{x^2+2x+2} + \frac{x-2}{(x^2+2x+2)^2}\right] dx$

$= \int \left[\frac{x+1}{(x+1)^2+1} + \frac{-1}{(x+1)^2+1} + \frac{x+1}{((x+1)^2+1)^2} + \frac{-3}{((x+1)^2+1)^2}\right] dx$

$= \frac{1}{2}\ln|(x+1)^2+1| - \tan^{-1}(x+1) + \frac{-1}{2}\frac{1}{(x+1)^2+1} + \int \frac{-3}{((x+1)^2+1)^2} dx$

$\int \frac{1}{((x+1)^2+1)^2} dx = \int \frac{\sec^2(\theta)}{\sec(\theta)} d\theta = \int \frac{1}{\sec^2(\theta)} d\theta \qquad (x+1 = \tan(\theta))$

$= \frac{1}{2}\int 1 + \cos(2\theta) d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) = \frac{1}{2}\tan^{-1}(x+1) + \frac{1}{4}\frac{2\tan(\theta)}{\tan^2(\theta)+1}$

$= \frac{1}{2}\tan^{-1}(x+1) + \frac{1}{2}\frac{x+1}{(x+1)^2+1}$

So answer =

$\frac{1}{2}\ln|(x+1)^2+1| - \tan^{-1}(x+1) + \frac{-1}{2}\frac{1}{(x+1)^2+1} + \frac{-3}{2}\tan^{-1}(x+1) + \frac{-3}{2}\frac{x+1}{(x+1)^2+1} + C$

$= \frac{1}{2}\ln|(x+1)^2+1| - \frac{5}{2}\tan^{-1}(x+1) + \frac{-3x-4}{(x+1)^2+1} + C$