# Section 7.3

4.

Let x=4sinθ,($\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$). Then , $dx = 4\cos\theta\, d\theta$ and $\sqrt{16-x^2} = \sqrt{16\cos^2\theta}$

$= 4|\cos\theta|$=4cosθ. When x=0, $4\sin\theta = 0 \Rightarrow$ θ=0.and when x=$2\sqrt{3} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$ . So we have $\int_0^{\frac{\sqrt{3}}{2}} \frac{x^3}{\sqrt{16-x^2}} dx = \int_0^{\frac{\pi}{3}} \frac{4^3 \sin^3\theta}{4\cos\theta} 4\cos\theta\, d\theta$

$= 64 \int_0^{\frac{\pi}{3}} (1-\cos^2\theta)\sin\theta\, d\theta$ = - 64 $\int_0^{\frac{\pi}{3}} (1-\cos^2\theta)\, d(\cos\theta)$= -64( $-\frac{5}{24}$)= $\frac{40}{3}$ ..

5.

Let t=secθ. Then dt= $\sec\theta\tan\theta d\theta$ , t=$\sqrt{2} \Rightarrow \theta = \frac{\pi}{4}$ ,t=2 $\Rightarrow \theta = \frac{\pi}{3}$.

So we have $\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}} dt = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{4\sec^3\theta\tan\theta} \sec\theta\tan\theta\, d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^2\theta\, d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1+\cos 2\theta}{2} d\theta$

$= \frac{1}{2}(\frac{\pi}{12} + \frac{\sqrt{3}}{4} - \frac{1}{2}) = \frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}$ .

9.

Let x=4tanθ ,($\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$). Then dx=$4\sec^2\theta d\theta$ and surely $\sqrt{x^2+16}$ = 4secθ.

So we have $\int \frac{dx}{\sqrt{x^2+16}} = \int \frac{4\sec^2\theta d\theta}{4\sec\theta}$ = $\ln|\sec\theta + \tan\theta|$+ c' = $\ln\left|\sqrt{x^2+16} + x\right| - \ln 4$+ c'

$= \ln\left|\sqrt{x^2+16} + x\right| + c$ .

14.

Let u=$\sqrt{5}\sin\theta$. Then, du = $\sqrt{5}\cos\theta\, d\theta$. So we have $\int \frac{du}{u\sqrt{5-u^2}} = \int \frac{\sqrt{5}\cos\theta\, d\theta}{\sqrt{5}\sin\theta\sqrt{5}\cos\theta}$

$= \frac{1}{\sqrt{5}} \int \csc\theta\, d\theta = \frac{1}{\sqrt{5}} \ln|\csc\theta - \cot\theta|$+c = $\frac{1}{\sqrt{5}} \ln\left|\frac{\sqrt{5}-\sqrt{5-u^2}}{u}\right|$+c..

16.

Let x= $\frac{\sec\theta}{3}$ .Then dx= $\frac{\sec\theta \cdot \tan\theta\, d\theta}{3}$ ,and x= $\frac{\sqrt{2}}{3} \Rightarrow \theta = \frac{\pi}{4}$, x=$\frac{2}{3} \Rightarrow \theta = \frac{\pi}{3}$ .So we have

$\int_{\frac{\sqrt{2}}{3}}^{\frac{2}{3}} \frac{dx}{x^5\sqrt{9x^2-1}} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{3^4\, d\theta}{\sec^4\theta}$ =81$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^4\theta\, d\theta$ =81$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\frac{1+\cos 2\theta}{2})^2 d\theta = \frac{81}{4}(\frac{\pi}{8} - \frac{7\sqrt{3}}{16}$ -1)..

22.

Let $x=\tan\theta, (\frac{-\pi}{2} \le \theta \le \frac{\pi}{2})$. Then $dx=\sec^2\theta d\theta$, $\sqrt{x^2+1} = \sec\theta$. And x=0

$\Rightarrow \theta = 0, x = 1 \Rightarrow \theta = \frac{\pi}{4}$. So we have $\int_0^1 \sqrt{x^2+1}dx = \int_0^{\frac{\pi}{4}} \sec\theta \sec^2\theta d\theta$

$= \frac{1}{2}[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|]_0^{\frac{\pi}{4}} = \frac{1}{2}[\sqrt{2} + \ln(1+\sqrt{2})]$..

35.

First the area of $\Delta POQ = \frac{1}{2}(r\cos\theta)(r\sin\theta) = \frac{1}{2}r^2\sin\theta\cos\theta$. And area of

$PQR = \int_{r\cos\theta}^{r} \sqrt{r^2-x^2}\, dx$. Let x= r cos(u) $\Rightarrow$ dx =-r sin(u) du ($u \in [0\ \frac{\pi}{2}]$)

So we get $\int \sqrt{r^2-x^2}dx = \int r\sin(u)(-r\sin(u))\, du = -\frac{1}{2}r^2\cos^{-1}(\frac{x}{r}) + \frac{1}{2}x\sqrt{r^2-x^2} + c$

Then the area of PQR = $\frac{1}{2}[-r^2\cos^{-1}(\frac{x}{r}) + x\sqrt{r^2-x^2}]_{r\cos\theta}^{r} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta\cos\theta$

Therefore, the area of sector POR = area $\Delta POQ$ + area PQR = $\frac{1}{2}r^2\theta$..

43.

I use cylindrical shells and let $R > r$ .Then equation $x^2 = r^2 - (y-R)^2$

$\Rightarrow x = \pm\sqrt{r^2-(y-R)^2}$ , so $h(y) = 2\sqrt{r^2-(y-R)^2}$ we have

$V = \int_{R-r}^{R+r} 2\pi y \cdot 2\sqrt{r^2-(y-R)^2}\, dy = \int_{-r}^{r} 4\pi(u+R) \cdot 2\sqrt{r^2-u^2}\, du$ $(u = y - R)$

$= 4\pi \int_{-r}^{r} u\sqrt{r^2-u^2}du + 4\pi R \int_{-r}^{r} \sqrt{r^2-u^2}du$

$= 4\pi[-\frac{1}{3}(r^2-u^2)^{\frac{2}{3}}]_{-r}^{r} + 4\pi R \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} r^2\cos^2\theta\, d\theta$ (since $u = r\sin\theta$)

$= 2\pi^2 R r^2$..