Section 7.2

2. $\int \sin^6 x \cos^3 x\,dx = \int \sin^6 x \cos^2 x \cos x\,dx = \int \sin^6 x\,(1-\sin^2 x)\cos x\,dx \overset{s}{=} \int u^6(1-u^2)\,du$

$= \int (u^6 - u^8)\,du = \frac{1}{7}u^7 - \frac{1}{9}u^9 + C = \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$

3. $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x\,dx = \int_{\pi/2}^{3\pi/4} \sin^5 x \cos^2 x \cos x\,dx = \int_{\pi/2}^{3\pi/4} \sin^5 x\,(1-\sin^2 x)\cos x\,dx \overset{s}{=} \int_1^{\sqrt{2}/2} u^5(1-u^2)\,du$

$= \int_1^{\sqrt{2}/2}(u^5 - u^7)\,du = \left[\frac{1}{6}u^6 - \frac{1}{8}u^8\right]_1^{\sqrt{2}/2} = \left(\frac{1/8}{6} - \frac{1/16}{8}\right) - \left(\frac{1}{6} - \frac{1}{8}\right) = -\frac{11}{384}$

10. $\int_0^\pi \cos^6\theta\,d\theta = \int_0^\pi (\cos^2\theta)^3\,d\theta = \int_0^\pi \left[\frac{1}{2}(1+\cos 2\theta)\right]^3 d\theta = \frac{1}{8}\int_0^\pi (1 + 3\cos 2\theta + 3\cos^2 2\theta + \cos^3 2\theta)\,d\theta$

$= \frac{1}{8}\left[\theta + \frac{3}{2}\sin 2\theta\right]_0^\pi + \frac{1}{8}\int_0^\pi \left[\frac{3}{2}(1+\cos 4\theta)\right]d\theta + \frac{1}{8}\int_0^\pi \left[(1-\sin^2 2\theta)\cos 2\theta\right]d\theta$

$= \frac{1}{8}\pi + \frac{3}{16}\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^\pi + \frac{1}{8}\int_0^0 (1-u^2)\left(\frac{1}{2}du\right) \qquad [u = \sin 2\theta,\ du = 2\cos 2\theta\,d\theta]$

$= \frac{\pi}{8} + \frac{3\pi}{16} + 0 = \frac{5\pi}{16}$

14. $\int_0^\pi \sin^2 t \cos^4 t\,dt = \frac{1}{4}\int_0^\pi (4\sin^2 t\cos^2 t)\cos^2 t\,dt = \frac{1}{4}\int_0^\pi (2\sin t\cos t)^2\,\frac{1}{2}(1+\cos 2t)\,dt$

$= \frac{1}{8}\int_0^\pi (\sin 2t)^2(1+\cos 2t)\,dt = \frac{1}{8}\int_0^\pi (\sin^2 2t + \sin^2 2t\cos 2t)\,dt$

$= \frac{1}{8}\int_0^\pi \sin^2 2t\,dt + \frac{1}{8}\int_0^\pi \sin^2 2t\cos 2t\,dt = \frac{1}{8}\int_0^\pi \frac{1}{2}(1-\cos 4t)\,dt + \frac{1}{8}\left[\frac{1}{3}\cdot\frac{1}{2}\sin^3 2t\right]_0^\pi$

$= \frac{1}{16}\left[t - \frac{1}{4}\sin 4t\right]_0^\pi + \frac{1}{8}(0-0) = \frac{1}{16}\left[(\pi - 0) - 0\right] = \frac{\pi}{16}$

15. $\displaystyle\int \frac{\cos^5\alpha}{\sqrt{\sin\alpha}}\,d\alpha = \int \frac{\cos^4\alpha}{\sqrt{\sin\alpha}}\cos\alpha\,d\alpha = \int \frac{(1-\sin^2\alpha)^2}{\sqrt{\sin\alpha}}\cos\alpha\,d\alpha \overset{s}{=} \int \frac{(1-u^2)^2}{\sqrt{u}}\,du$

$\displaystyle= \int \frac{1-2u^2+u^4}{u^{1/2}}\,du = \int (u^{-1/2} - 2u^{3/2} + u^{7/2})\,du = 2u^{1/2} - \frac{4}{5}u^{5/2} + \frac{2}{9}u^{9/2} + C$

$= \frac{2}{45}u^{1/2}(45 - 18u^2 + 5u^4) + C = \frac{2}{45}\sqrt{\sin\alpha}\,(45 - 18\sin^2\alpha + 5\sin^4\alpha) + C$

23. $\int \tan^2 x\,dx = \int (\sec^2 x - 1)\,dx = \tan x - x + C$

29. $\int \tan^3 x\sec x\,dx = \int \tan^2 x\sec x\tan x\,dx = \int (\sec^2 x - 1)\sec x\tan x\,dx$

$= \int (u^2-1)\,du \quad [u = \sec x,\ du = \sec x\tan x\,dx] \quad = \frac{1}{3}u^3 - u + C = \frac{1}{3}\sec^3 x - \sec x + C$

43. $\int \sin 8x\cos 5x\,dx \overset{2a}{=} \int \frac{1}{2}[\sin(8x-5x) + \sin(8x+5x)]\,dx = \frac{1}{2}\int \sin 3x\,dx + \frac{1}{2}\int \sin 13x\,dx$

$= -\frac{1}{6}\cos 3x - \frac{1}{26}\cos 13x + C$