

[7.1] Integration by Parts

$$3. \int x \cos 5x dx = \int x d(\frac{1}{5} \sin 5x) = x \frac{1}{5} \sin 5x - \int \frac{1}{5} \sin 5x dx = x \frac{1}{5} \sin 5x - \int \frac{1}{25} \sin 5x d5x = x \frac{1}{5} \sin 5x + \frac{1}{25} \cos 5x + C.$$

$$10. \int \sin^{-1} x dx = \sin^{-1} x \dot{x} - \int x d \sin^{-1} x = \sin^{-1} x \dot{x} - \int x \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x \dot{x} - \int (1-x^2)^{-1/2} \frac{1}{2} dx^2 = \sin^{-1} x \dot{x} + (1-x^2)^{1/2} + C.$$

$$12. \int p^5 \ln p dp = \int \ln p d(\frac{1}{6} p^6) = \ln p \frac{1}{6} p^6 - \int \frac{1}{6} p^6 \frac{1}{p} dp = \frac{1}{6} p^6 \ln p - \int \frac{1}{6} p^5 dp = \frac{1}{6} p^6 \ln p - \frac{1}{36} p^6 + C.$$

$$13. \int t \sec^2 2t dt = \int t \frac{1}{2} d \tan 2t = \frac{1}{2} t \tan 2t - \frac{1}{2} \int \tan 2t dt = \frac{1}{2} t \tan 2t - \frac{1}{4} \int \tan 2t d2t = \frac{1}{2} t \tan 2t - \frac{1}{4} \ln |\sec 2t| + C.$$

$$15. \int (\ln x)^2 dx = (\ln x)^2 x - \int x d(\ln x)^2 = x(\ln x)^2 - \int x 2(\ln x) \frac{1}{x} dx = x(\ln x)^2 - 2 \int \ln x dx = x(\ln x)^2 - 2(x \ln x - \int x d(\ln x)) = x(\ln x)^2 - 2(x \ln x - \int 1 dx) = x(\ln x)^2 - 2(x \ln x - x + C).$$

$$25. \int_0^1 \frac{y}{e^{2y}} dy = \int_0^1 y e^{-2y} dy = \frac{1}{-2} \int_0^1 y d e^{-2y} = \frac{1}{-2} (y e^{-2y} |_0^1 - \int_0^1 e^{-2y} dy) = \frac{1}{-2} (e^{-2} - \frac{1}{-2} e^{-2y} |_0^1) = \frac{1}{-2} (e^{-2} + \frac{1}{2} e^{-2} - \frac{1}{2}) = \frac{1}{4} - \frac{3}{4} e^{-2}.$$

$$29. \int \cos x \ln(\sin x) dx = \int \ln(\sin x) d(\sin x) = (\sin x) \ln(\sin x) - \int \sin x d \ln(\sin x) = (\sin x) \ln(\sin x) - \int \cos x dx = (\sin x) \ln(\sin x) - \sin x + C.$$

$$35. \text{ Let } x = \theta^2. \text{ Then } \int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2) d\theta = \frac{1}{2} \int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^2 \cos(\theta^2) d\theta^2 = \frac{1}{2} \int_{\pi/2}^{\pi} x \cos x dx = \frac{1}{2} \int_{\pi/2}^{\pi} x d \sin x = \frac{1}{2} (x \sin x |_{{\pi/2}}^{\pi} - \int_{\pi/2}^{\pi} \sin x dx) = -\frac{\pi}{4} - \frac{1}{2} [-\cos x]_{\pi/2}^{\pi} = -\frac{\pi}{4} - \frac{1}{2}.$$

$$47. \int (\ln x)^n dx = (\ln x)x - \int x d(\ln x)^n = (\ln x)x - \int x n(\ln x)^{n-1} \frac{1}{x} dx = (\ln x)x - n \int (\ln x)^{n-1} dx.$$

$$53. \int_0^5 x e^{-0.4x} dx = \frac{1}{-0.4} \int_0^5 x d e^{-0.4x} = \frac{1}{-0.4} \left(x e^{-0.4x} |_0^5 - \int_0^5 e^{-0.4x} dx \right) = \frac{5}{-2} \left(5 e^{-2} - \frac{5}{-2} e^{-0.4x} |_0^5 \right) = -\frac{25}{2} 5 e^{-2} - \frac{25}{4} e^{-2} + \frac{25}{4}.$$