[Section 6.3] Volumes by cylindrical shells

28. Midpoint rule:

 $\int_a^b x f(x) \ dx \approx \sum x_i f(\overline{x}_i) \Delta x \ , \ \text{where} \ \Delta x = \frac{b-a}{n} \ \text{and} \ \overline{x}_i = \frac{1}{2} (\overline{x}_{i-1} + \overline{x}_i) \ , \ i = 1, 2, ..., n \in \mathbb{N}$

46. By symmetry, the volume of a napkin ring obtained by drilling a hole of radius r through a sphere with radius R is twice the volume obtained by rotating the area above the x-axis and below the curve $y = \sqrt{R^2 - x^2}$ (the equation of the top half of the cross-section of the sphere), between x = r and x = R, about the y-axis.