[Section 6.3] Volumes by cylindrical shells

1. $V = \int_0^1 2\pi x f(x)\ dx = \int_0^1 2\pi(x^4 - 2x^3 + x^2)\ dx = 2\pi[\frac{1}{5} - \frac{1}{2} + \frac{1}{3}] = \frac{\pi}{15}$ ■

2. $V = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\ dx = \pi \int_0^{\pi} \sin u\ du = \pi(-\cos u\big|_0^{\pi}) = 2\pi$ ■

10. $V = \int_0^1 2\pi y\sqrt{y}\ dy = \int_0^1 2\pi y^{\frac{3}{2}}\ dy = \frac{4\pi}{5} y^{\frac{5}{2}}\big|_0^1 = \frac{4\pi}{5}$ ■

16. $V = \int_0^1 2\pi(x+1)\sqrt{x}\ dx = 2\pi \int_0^1 x^{\frac{3}{2}} + x^{\frac{1}{2}}\ dx = 2\pi[\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}]_0^1 = 2\pi[\frac{2}{5} + \frac{2}{3}] = \frac{32}{15}\pi$ ■

17. $V = \int_1^3 2\pi(x-1)(-x^2+4x-3)\ dx = 2\pi \int_1^3(-x^3+5x^2-7x+3)\ dx = 2\pi[-\frac{1}{4}x^4+\frac{5}{3}x^3-\frac{7}{2}x^2+3x]_1^3$
$= \frac{44}{3}\pi - 12\pi = \frac{8}{3}\pi$ ■

21. $V = \int_1^2 2\pi x \ln x\ dx$ ■

28. $n = 5\ ,\ a = 2\ ,\ b = 12 \quad \Rightarrow \quad \Delta x = \frac{12-2}{5} = 2$
$V = \int_2^{12} 2\pi x f(x)\ dx \approx 2\pi[\sum x_i f(\overline{x}_i)\Delta x] = 2\pi[3f(3) + 5f(5) + 7f(7) + 9f(9) + 11f(11)]\ 2$
$\approx 4\pi[3\cdot 2 + 5\cdot 4 + 7\cdot 4 + 9\cdot 2 + 11\cdot 1] = 332\pi$ ■

46. By symmetry, the volume of a napkin ring obtained by drilling a hole of radius r through a sphere with radius R is twice the volume obtained by rotating the area above the x-axis and below the curve $y = \sqrt{R^2 - x^2}$ (the equation of the top half of the cross-section of the sphere),between $x = r$ and $x = R$,about the y-axis.

$$V = 2\int_r^R 2\pi x\sqrt{R^2 - x^2}\ dx = 4\pi[-\frac{1}{3}(R^2 - r^2)^{3/2}]_r^R = \frac{4}{3}\pi(R^2 - r^2)^{3/2} = \frac{\pi}{6}h^3$$

since $(\frac{h}{2})^2 = R^2 - r^2$ ■