Solutions of 6.2

7. A cross-section is a washer with inner radius x^3 and outer radius x, so its area is

$$A(x) = \pi(x)^2 - \pi(x^3)^2 = \pi(x^2 - x^6).$$

$$V = \int_0^1 A(x)dx = \int_0^1 \pi(x^2 - x^6)dx = \pi\left[\frac{1}{3}x^3 - \frac{1}{7}x^7\right]_0^1 = \pi(\frac{1}{3} - \frac{1}{7}) = \frac{4}{21}\pi.$$

11. A cross-section is a washer with inner radius $1 - \sqrt{x}$ and outer radius $1 - x$, so its area is

$$A(x) = \pi(1-x)^2 - \pi(1-\sqrt{x})^2 = \pi(-3x + x^2 + 2\sqrt{x}).$$

$$V = \int_0^1 A(x)dx = \pi\left[-\frac{3}{2}x^2 + \frac{1}{3}x^3 + \frac{4}{3}x^{\frac{3}{2}}\right]_0^1 = \pi(-\frac{3}{2} + \frac{1}{3} + \frac{4}{3}) = \frac{\pi}{6}.$$

20.

$$V = \int_0^1 A(y)dy = \int_0^1 [\pi(1)^2 - \pi(\sqrt[3]{y})^2]dy = \pi\left[y - \frac{3}{5}y^{\frac{5}{3}}\right]_0^1 = \frac{2}{5}\pi.$$

21.

$$V = \int_0^1 A(y)dy = \int_0^1 [\pi(1 - \sqrt[3]{y})^2]dy = \pi\left[y - \frac{3}{2}y^{\frac{4}{3}} + \frac{3}{5}y^{\frac{5}{3}}\right]_0^1 = \frac{\pi}{10}.$$

22.

$$V = \int_0^1 A(x)dx = \int_0^1 [\pi(1)^2 - \pi(1-x^3)^2]dy = \frac{5}{14}\pi.$$

31. The curve $y = \tan^3 x$ intersects $y = 1$ at $(\frac{\pi}{4}, 1)$. So

$$V = \int_0^{\frac{\pi}{4}} \pi(1 - \tan^3 x)^2 dx.$$

32. $y = (x-2)^4$ and $8x - y = 16$ intersect at $(2, 0)$ and $(4, 16)$. So

$$V = \int_0^{16} \pi\left\{\left[10 - (\frac{y}{8} + 2)\right]^2 - [10 - (2 + \sqrt[4]{y})]^2\right\} dy$$

71. (a) The radius of the end $x = \frac{h}{2}$ is $R - c\cdot(\frac{h}{2})^2$, and the radius of the end $x = -\frac{h}{2}$ is $R - c\cdot(-\frac{h}{2})^2$, and each is equal to $R - d$, where $d = \frac{ch^2}{4}$.

(b)

$$V = \int_{-\frac{h}{2}}^{\frac{h}{2}} \pi(R - cx^2)^2 dx$$

Direct calculation shows that

$$V = \frac{1}{3}\pi h(2R^2 + r^2 - \frac{2}{5}d^2).$$