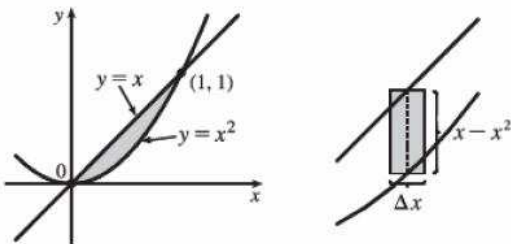


## Section 6.1

$$(2). A = \int_0^2 (\sqrt{x+2} - \frac{1}{x+1}) dx = [\frac{2}{3}(x+2)^{\frac{3}{2}} - \ln(x+1)]_0^2 = \frac{16-4\sqrt{2}}{3} - \ln 3.$$

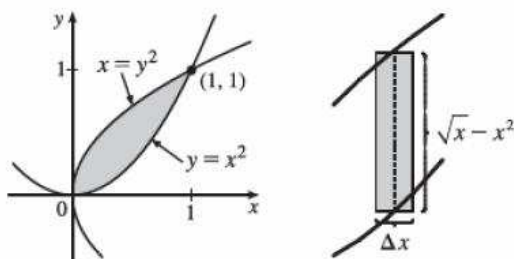
$$(3). A = \int_{-1}^1 [e^y - (y^2 - 2)] dy = [e^y - \frac{1}{3}y^3 + 2y]_{-1}^1 = e - e^{-1} + \frac{10}{3}.$$

(7). The curves intersect when  $x = x^2 \Leftrightarrow x = 0$  or  $1$ .



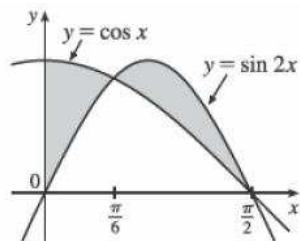
$$A = \int_0^1 (x - x^2) dx = [\frac{1}{2}x^2 - \frac{1}{3}x^3]_0^1 = \frac{1}{6}.$$

(11). The curves intersect at  $(0,0)$  and  $(1,1)$ .



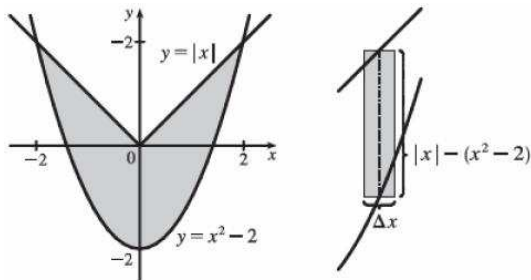
$$A = \int_0^1 (\sqrt{x} - x^2) dx = [\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3]_0^1 = \frac{1}{3}.$$

(23). The curves intersect at  $\cos x = \sin 2x = 2 \sin x \cos x \Leftrightarrow x = \frac{\pi}{6}$  or  $\frac{\pi}{2}$ .



$$A = \int_0^{\frac{\pi}{6}} (\cos x - \sin 2x) dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x - \cos x) dx \\ = [\sin x + \frac{1}{2} \cos 2x]_0^{\frac{\pi}{6}} + [-\frac{1}{2} \cos 2x - \sin x]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{1}{2}.$$

(26). For  $x > 0$ , curves intersect when  $x = x^2 - 2 \Rightarrow x = 2$ , for  $x < 0$ , curves intersect when  $-x = x^2 - 2 \Rightarrow x = -2$ .



$$A = \int_{-2}^0 (-x - x^2 + 2) dx + \int_0^2 (x - x^2 + 2) dx = [-\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x]_{-2}^0 + [\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x]_0^2 = \frac{20}{3}.$$

(42).  $\Delta x = 2$ , thus

$$A \approx 2 \cdot \left[ \frac{0+6.2}{2} + \frac{6.2+7.2}{2} + \frac{7.2+6.8}{2} + \frac{6.8+5.6}{2} + \frac{5.6+5.0}{2} + \frac{5.0+4.8}{2} + \frac{4.8+4.8}{2} + \frac{4.8+0}{2} \right] = 80.8(\text{m}^2).$$

(45). The area under curve  $A$  between  $t = 0$  and  $t = x$  is  $\int_0^x v_A(t)dt = s_A(x)$ , where  $v_A$  is the velocity of car  $A$  and  $s_A$  is its displacement. Similarly for car  $B$ .

- a. After one minute, the area under curve  $A$  is greater than the area under curve  $B$ , hence  $A$  is ahead.
- b. The area of the shaded region is  $s_A(x) - s_B(x)$ , which is the distance of two cars after one minute.
- c. Since the area of the orange region is greater than that of the white one, car  $A$  is still ahead.
- d. In the first minute, the distance by which car  $A$  is ahead, seems to be about 3 squares. We estimate the time  $x$  such that the area between the curves for  $1 \leq t \leq x$  is the same as the area for  $0 \leq t \leq 1$ . From the graph,  $x \approx 2.2$ , so the two cars are side by side at  $t \approx 2.2$  minutes.