3. $\frac{2}{9}(x^3+1)^{\frac{3}{2}}+c$

let $u = x^3+1$, then $\frac{du}{dx} = 3x^2$
$\int x^2\sqrt{x^3+1}dx = \int x^2\sqrt{u}\frac{1}{3x^2}du = \int \frac{1}{3}\sqrt{u}du = \frac{2}{9}u^{\frac{3}{2}} + c = \frac{2}{9}(x^3+1)^{\frac{3}{2}}+c$

10. $\frac{1}{10.2}(3t+2)^{3.4}+c$

let $u = 3t+2$, then $\frac{du}{dt} = 3$
$\int(3t+2)^{2.4}dt = \int u^{2.4}\frac{1}{3}du = \frac{1}{10.2}u^{3.4}+c = \frac{1}{10.2}(3t+2)^{3.4}+c$

29. $e^{tanx}+c$

let $u = tanx$, then $\frac{du}{dx} = sec^2x$
$\int e^{tanx}sec^2xdx = \int e^u sec^2x\frac{1}{sec^2x}du = \int e^u du = e^u + c = e^{tanx}+c$

36. $tan^{-1}(cosx)+c$

let $u = cosx$, then $\frac{du}{dx} = -sinx$
$\int \frac{sinx}{1+cos^2x}dx = \int \frac{sinx}{1+u^2}\frac{1}{-sinx}du = \int \frac{-1}{1+u^2}du = tan^{-1}u + c = tan^{-1}(cosx)+c$

68. $\frac{1}{72}\pi^2$

let $u = sin^{-1}x$, then $\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$
$\int_0^{\frac{1}{2}} \frac{sin^{-1}x}{\sqrt{1-x^2}}dx = \int_0^{\frac{\pi}{6}} \frac{u}{\sqrt{1-x^2}}\sqrt{1-x^2}du = \int_0^{\frac{\pi}{6}} udu = \frac{1}{2}u^2|_0^{\frac{\pi}{6}} = \frac{1}{72}\pi^2$

69. $ln(e+1)$

let $u = e^z + z$, then $\frac{du}{dz} = e^z+1$
$\int_0^1 \frac{e^z+1}{e^z+z}dz = \int_1^{e+1} \frac{e^z+1}{u}\frac{1}{e^z+1}du = \int_1^{e+1}\frac{1}{u}du = ln(u)|_1^{e+1} = ln(e+1)$

80. $10000 - \frac{500000}{84}$

$\int_2^4 \frac{dx}{dt}dt = \int_2^4 5000(1-\frac{100}{(t+10)^2})dt = (5000t + 500000(t+10)^{-1})|_2^4 = 10000 - \frac{500000}{84}$

86.

$\int_0^\pi xf(sinx)dx$, let $\pi - x = y$, then $\frac{dy}{dx} = -1$
$= \int_\pi^0(\pi-y)f(sin(\pi-y))(-dy)$
$= \int_0^\pi(\pi-y)f(sin(\pi-y))dy$
$= \pi\int_0^\pi f(sin(\pi-y))dy - \int_0^\pi yf(sin(\pi-y))dy$
$= \pi\int_0^\pi f(siny)dy - \int_0^\pi yf(siny)dy$
$= \pi\int_0^\pi f(sinx)dx - \int_0^\pi xf(sinx)dx$
$\Longrightarrow 2\int_0^\pi xf(sinx)dx = \pi\int_0^\pi f(sinx)dx \Longrightarrow \int_0^\pi xf(sinx)dx = \frac{\pi}{2}\int_0^\pi f(sinx)dx$