Section 5.3

2. (a) $\int_0^0 f(t)dt = 0$, $\int_0^1 f(t)dt = \frac{1}{2}$, $\int_0^3 f(t)dt = -\frac{1}{2}$,$\int_0^2 f(t)dt = 0$, $\int_0^4 f(t)dt = 0$, $\int_0^5 f(t)dt = \frac{1}{2}$, $\int_0^{f(t)dt} = 1$

   (b) $\int_0^7 f(t)dt \doteqdot 1 + \frac{1}{2} \cdot 3 = \frac{2}{5}$

   (c) Maximum occurs at $t = 7$, minimal occurs at $t = 3$.

   (d) Ask your TA , if you want know details.

6. (i) $\frac{dg(x)}{dx} = \frac{d}{dx}(\int_0^x (1 + \sqrt{t})dt) = 1 + \sqrt{x}$

   (ii) $g(x) = \int_0^x 1 + \sqrt{t}dt = x + \frac{2}{3}x^{\frac{3}{2}} \implies \frac{dg(x)}{dx} = 1 + x^{\frac{1}{2}} = 1 + \sqrt{x}$

12. $G(x) = \int_x^1 \cos(\sqrt{t})dt \implies \frac{dG(x)}{dx} = \frac{d}{dx}(-\int_1^x \cos(\sqrt{t})dt) = -\cos(\sqrt{x})$

14. $h(x) = \int_0^{x^2} \sqrt{1 + r^3}dr \implies \frac{dh(x)}{dx} = \frac{d}{dx}\int_0^u \sqrt{1 + r^3}dr($ where $u = x^2)$
$= \frac{dx}{du}\frac{d}{du}\int_0^u \sqrt{1 + r^3}dr = 2x \cdot (\sqrt{1 + u^3}) = 2x\sqrt{1 + x^6}$

19. $\int_{-1}^2 (x^3 - 2x)dx = \frac{1}{4}x^4 - x^2|_{-1}^2 = \frac{3}{4}$

40. $\int_1^2 \frac{4 + u^2}{u^3}du = \int_1^2 \frac{4}{u^3} + \frac{1}{u}du = \ln u - u^{-4} = \ln 2 - \frac{1}{16}$

46. Since $\lim_{x \to \frac{\pi}{2}} = \infty$, hence this integral is a improper integral, we need detail it with more technique.

54. Let $u = x^2$, $v = \tan x$, then $\frac{dg}{dx} = \frac{du}{dx}\frac{d}{du}(\int_0^u \frac{dt}{\sqrt{2 + t^4}}) - \frac{dv}{dx}\frac{d}{dv}(\int_0^v \frac{dt}{\sqrt{2 + t^4}})$
$= \frac{2x}{\sqrt{2 + x^8}} - \frac{\sec^2 x}{\sqrt{1 + \tan^4 x}}$

63. (a) Local max occurs at $x = 9$, $x = 1$, $x = 5$, local min occurs at $x = 7$, $x = 3$.

   (b) Absolute max occurs at $x = 9$, absolute min occurs at $x = 3$.

   (c) $(\frac{1}{2}, 2)$, $(4, 6)$, $(8, 9)$

   (d) See page A90 in our text book.

66. $\lim_{n \to \infty} \frac{1}{n}(\sum_{i=1}^n (\sqrt{(\frac{1}{i})})) = \int_0^1 \sqrt{x}dx = \frac{2}{3}x^{\frac{3}{2}}|_0^1 = \frac{2}{3}$

71. (i) Since $\frac{x^2}{x^4 + x^2 + 1} \geq 0 \implies \int_5^{10} \frac{x^2}{x^4 + x^2 + 1}dx \geq 0$

   (ii) $\frac{x^2}{x^4 + x^2 + 1} \leq \frac{x^2}{x^4} \implies \int_5^{10} \frac{x^2}{x^4 + x^2 + 1}dx \leq \int_5^{10} \frac{x^2}{x^4}dx = \frac{1}{10} = 0.1$