1 Section 5.2

2.$f(x) = \ln x - 1, 1 \leq x \leq 4, \Delta x = \frac{4-1}{6} = \frac{1}{2}$

Since we are using left endpoints $x_i^* = x_{i-1}$

$L_6 = \sum_{i=1}^{n} f(x_{i-1})\Delta x$

$= (\Delta x)[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$

$= \frac{1}{2}[f(1) + f(\frac{3}{2}) + f(2) + f(\frac{5}{2}) + f(3) + f(\frac{(}{7})2)]$

$= \frac{1}{2}[(-1) + \ln\frac{3}{2} - 1 + \ln 2 - 1 + \ln\frac{5}{2} - 1 + \ln 3 - 1 + \ln\frac{7}{2} - 1]$

$= \frac{1}{2}(-6 + \ln\frac{105}{4})$

The Reimann sum represents the sum of the areas of the rectangles above the x-axis minus the sum of the areas of the rectangles below the x-axis ; that is , the bet area of the rectangles with respect to the x-axis.

5.$\Delta x = \frac{b-a}{n} = \frac{8-0}{4} = 2$

(a)Using the right endpoints to approximate $\int_0^8 f(x)dx$,we have

$\sum_{i=1}^{4} f(x_i)\Delta x = 2[f(2) + f(4) + f(6) + f(8)] \approx 4$

(b)Using the left endpoints to approximate $\int_0^8 f(x)dx$,we have

$\sum_{i=1}^{4} f(x_{i-1})\Delta x = 2[f(0) + f(2) + f(4) + f(6)] \approx 6$

(c)Using the midpoint of each subinterval to approximate $\int_0^8 f(x)dx$,we have

$\sum_{i=1}^{4} f(x_i)\Delta x = 2[f(1) + f(3) + f(5) + f(7)] \approx 10$

8. $\Delta x = \frac{(}{b} - a)n = \frac{9-3}{3} = 2$

(a)Using the right endpoints to approximate $\int_3^9 f(x)dx$,we have

$\sum_{i=1}^{3} f(x_i)\Delta x = 2[f(5) + f(7) + f(9)] = 2(-0.6 + 0.9 + 1.8) = 4.2$

(b)Using the left endpoints to approximate $\int_3^9 f(x)dx$,we have

$\sum_{i=1}^{3} f(x_{i-1})\Delta x = 2[f(3)+f(5)+f(7)] = 2(-3.4-0.6+0.9) = -6.2$

(c)Using the midpoint of each subinterval to approximate $\int_3^9 f(x)dx$,we have

$\sum_{i=1}^{3} f(x_i)\Delta x = 2[f(4)+f(6)+f(8)] = 2(-2.1+0.3+1.4) = -0.8$

We can not say anything about the midpoint compared to the exact value of the integral.

9.$\Delta x = \frac{10-2}{4} = 2$,so the endpoints are $2, 4, 6, 8, 10$ and the midpoints are $3, 5, 7, 9$.

The Midpoint Rule gives $\int^1 0_2 \sqrt{x^3+1}dx \approx \sum_{i=1}^{4} f(x_i)\Delta x = 2(\sqrt{3^3+1} + \sqrt{5^3+1} + \sqrt{7^3+1} + \sqrt{9^3+1}) \approx 124.1644$

14. See the solution to Exercise5.1.7 for a possible algorithm to calculate the sums. With $\Delta x = \frac{1-0}{100} = 0.01$ and subinterval endpoints 1,1.01,1.02,...,1.99,2, we calculate that the left Riemann sum is $L_{100} = \sum_{i=1}^{100} \sin(x_{i-1}^2)\Delta x \approx 0.30607$, and the right Riemann sum is $R_{100} = \sum_{i=1}^{100} \sin(x_i^2)\Delta x \approx 0.31448$.

Since $f(x) = \sin(x^2)$ is an increasig function, we have $L_{100} \le \int_0^1 \sin(x^2)dx \le R_{100}$, so $0.306 < L_{100} \le \int_0^1 \sin(x^2)dx \le R_{100} < 0.315$.

Therefore, the approximate value 0.3084$\approx$ 0.31 in Exercise must be accurate to two decimal places.

18. On $[\pi, 2\pi]$, $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos x_i}{x_i}\Delta x = \int_{\pi}^{2\pi} \frac{\cos x}{x}dx$

30. $\Delta x = \frac{10-1}{n} = \frac{9}{n}$ and $x_i = 1 + i\Delta x = 1 + \frac{9i}{n}$, so

$\int^1 0_1(x - 4\ln x)dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n}[(1+\frac{9i}{n}) - 4\ln(1+\frac{9i}{n})]\frac{9}{n}$

33.

(a) Think of $\int_0^2 f(x)dx$ as the area of a trapezoid with bases 1 and 3 and height 2. The area of a trapezoid is $A = \frac{1}{2}(b+B)h$, so $\int_0^2 f(x)dx = \frac{1}{2}(1+3)2 = 4$.

(b)$\int_0^5 f(x)dx = \int_0^2 f(x)dx + \int_2^3 f(x)dx + \int_3^5 f(x)dx =$trapezoid + rectangle + triangle $= \frac{1}{2}(1+3)2 + 3\dot{1} + \frac{1}{2}\dot{2}\dot{3} = 4+3+3 = 10$

(c)$\int_5^7 f(x)dx$ is the negative of the area of the triangle with base 2 and height 3. $\int_5^7 f(x)dx = \frac{-1}{2}\dot{2}\dot{3} = -3$.

(d) $\int_7^9 f(x)dx$ is the negative of the area of a trapezoid with base 3 and 2 and height 2 , so it equals $-\frac{1}{2}(B+b)h = -\frac{1}{2}(3+2)2 = -5$. Thus, $\int_0^9 f(x)dx = \int_0^5 f(x)dx + \int_5^7 f(x)dx + \int_7^9 f(x)dx = 10 + (-3) + (-5) = 2$.

36. $\int_{-2}^2 \sqrt{4-x^2}dx$ can be interpreted as the area under the graph of $f(x) = \sqrt{4-x^2}$ between $x=-2$ and $x=2$. This is equal to half the area of the circle with radius 2 , so $\int_{-2}^2 \sqrt{4-x^2}dx = \frac{1}{2}\pi\dot{2}^2 = 2\pi$.

41.$\int_\pi^\pi \sin^2 x \cos^4 x dx = 0$ since the limits of integration are equal.

47.$\int_{-2}^2 f(x)dx + \int_2^5 f(x)dx - \int_{-2}^{-1} f(x)dx = \int_{-2}^5 f(x)dx + \int_{-1}^{-2} f(x)dx = \int_{-1}^5 f(x)dx$

66. Since $|\sin 2x| \leq 1$, $|\int_0^{2\pi} f(x)\sin 2x dx| \leq \int_0^{2\pi} |f(x)\sin 2x| dx \leq \int_0^{2\pi} |f(x)||\sin 2x| dx \leq \int_0^{2\pi} |f(x)| dx$

70. It's an n-partition on $[0,1]$,so $\frac{1-0}{n} = \frac{1}{n}$, and let $f(x) = \frac{1}{1+x^2}$ and then $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^n \frac{1}{1+\frac{i}{n}^2} = \int_0^1 f(x)dx = \int_0^1 \frac{1}{1+x^2}dx$