13. *sol.*

The antiderivative of $f(u) = \dfrac{u^4 + 3\sqrt{u}}{u^2} = u^2 + 3u^{-3/2}$ is $F(u) = \dfrac{1}{3}u^3 - 6u^{-1/2} + C$, where C is a constant.

14. *sol.*

The antiderivative of $f(u) = 3e^x + 7\sec^2 x$ is $F(u) = 3e^x + 7\tan x + C$, where C is a constant.

24. *sol.*

$f''(x) = 2 + x^3 + x^6$, $f'(x) = C_1 + 2x + \dfrac{1}{4}x^4 + \dfrac{1}{7}x^7$, $f(x) = C_2 + C_1 x + x^2 + \dfrac{1}{20}x^5 + \dfrac{1}{56}x^8$, where C_1, C_2 are constants.

40. *sol.*

$f''(t) = \dfrac{3}{\sqrt{t}}$, $f'(t) = C_1 + 6\sqrt{t}$, $f(t) = C_2 + C_1 t + 4t^{3/2}$. Substitute $f(4) = 20$ and $f'(4) = 7$ get $C_1 = -5$ and $C_2 = 8$, so $f(t) = 4t^{3/2} - 5t + 8$.

46. *sol.*

$f'''(x) = \cos x$, $f''(x) = C_1 + \sin x$, $f'(x) = C_2 + C_1 x - \cos x$, $f(x) = C_3 + C_2 x + \dfrac{1}{2}C_1 x^2 - \sin x$. Substitute $f(0) = 1$, $f'(0) = 2$ and $f''(0) = 2$ get $C_1 = 3$, $C_2 = 3$, $C_3 = 1$, so $f(x) = -\sin x + \dfrac{3}{2}x^2 + 3x + 1$.

52. *sol.*

Assume that the motion: constant positive acceleration → constant velocity → constant negative acceleration → at rest ($v = 0$) → constant negative acceleration → constant positive acceleration → at rest ($v = 0$).

Suppose $x(0) = p_0$. At time t, $x(t)$ equals p_0 plus the displacement of the particle, where the displacement is the area between velocity function and x-axis.

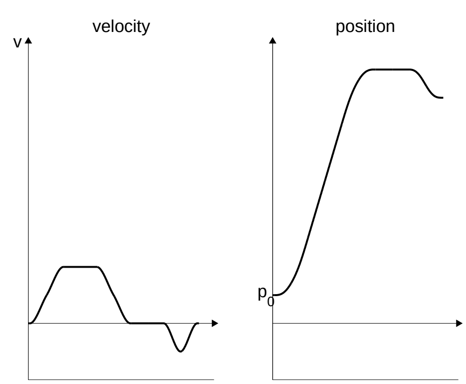

Figure 1: 4.9 Ex.52

56. *sol.*

$f(x) = \sqrt{x^4 - 2x^2 + 2} - 1 = \sqrt{(x^2-1)^2 + 1} - 1$, so $f(1) = f(-1) = 0$ and $f(x) \geq 0$ for all $x \in [-1.5, 1.5]$.

$f'(x) = \dfrac{2x(x-1)(x+1)}{\sqrt{x^4 - 2x^2 + 2}}$, so we can determine the sign of $f'(x)$ in $[-1.5, 1.5]$.

Let $F(x)$ be the antiderivative of $f(x)$ with $F(0) = 0$:

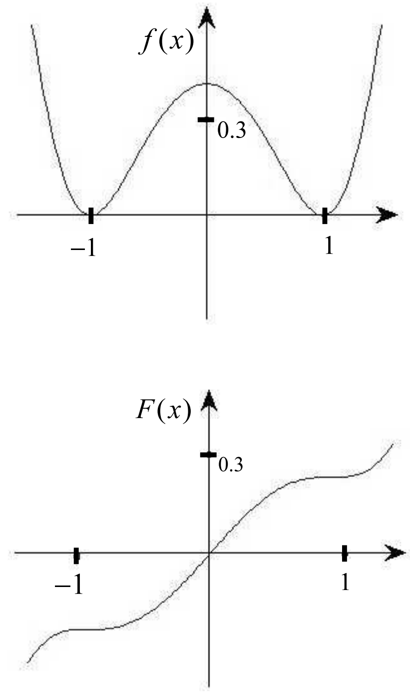

Figure 2: 4.9 Ex.56

68. *sol.*

(a)$EIy'' = mg(L-x) + \dfrac{1}{2}\rho g(L-x)^2$

$$\Rightarrow y'' = \frac{mg}{EI}(L-x) + \frac{\rho g}{2EI}(L-x)^2$$

$$\Rightarrow y' = \frac{mg}{EI} \cdot \frac{-1}{2}(L-x)^2 + \frac{\rho g}{2EI} \cdot \frac{-1}{3}(L-x)^3 + C$$

$$\Rightarrow y = \frac{mg}{EI} \cdot \frac{1}{6}(L-x)^3 + \frac{\rho g}{2EI} \cdot \frac{1}{12}(L-x)^4 + Cx + D$$

Substitute $y(0) = 0$ and $y'(0) = 0$ and get

$$y = \frac{mg}{EI} \cdot \frac{1}{6}(L-x)^3 + \frac{\rho g}{2EI} \cdot \frac{1}{12}(L-x)^4 + \left(\frac{3mgL^2 + \rho g L^3}{6EI}\right)x - \left(\frac{4mgL^3 + \rho g L^4}{24EI}\right)$$

(b) $f(L) = \dfrac{8mgL^3 + 3\rho g L^4}{24EI}$

72. *sol.*

Let $a(t)$, $v(t)$, and $f(t)$ be the acceleration function, velocity function and position function. Hence we have $v(0) = 80(\text{km/hr}) = \frac{200}{9}(\text{m/sec})$, and $a(t) = -7(\text{m/sec}^2)$. Then $v(t) = -7t + C$, substitute by $v(0) = \frac{200}{9}$ get $C = \frac{200}{9}$. So $v(t) = -7t + \frac{200}{9}$. Then $f(t) = -\frac{7}{2}t^2 + \frac{200}{9}t + D$. Let $v(t) = 0$, then $t = \frac{200}{63}$. The distance traveled by the car before it stops is $f(\frac{200}{63}) - f(0) = \frac{20000}{567} \approx 35.27(\text{m})$.

76. *sol.*

(a) Assume the acceleration of gravity $g = -10(\text{m/s})$. So we have

$$a(t) = \begin{cases} 18t & 0 \le t \le 3 \\ -10 & 3 \le t \le 17 \\ k & 17 \le t \le 22 \\ 0 & t \ge 22 \end{cases}$$

and $v(0) = 0$, $v(22) = -5.5$ (Note: The velocity of the rocket (downward) slows linearly between $t = 17$ to $t = 22$, so its acceleration is a constant during the five seconds.) Find the antiderivative of $a(t)$, we have the velocity function of the rocket:

$$v(t) = \begin{cases} 9t^2 + C_1 & 0 \le t \le 3 \\ -10t + C_2 & 3 \le t \le 17 \\ kt + C_3 & 17 \le t \le 22 \\ C_4 & t \ge 22 \end{cases}$$

Then $v(0) = 0 \Rightarrow C_1 = 0$, $v(3) = 81$; $v(3) = 81 \Rightarrow C_2 = 111$, $v(17) = -59$; $v(17) = -59$ and $v(22) = -5.5 \Rightarrow k = 10.7$, $C_3 = -240.9$, $C_4 = -5.5$. So,

$$v(t) = \begin{cases} 9t^2 & 0 \le t \le 3 \\ -10t + 111 & 3 \le t \le 17 \\ 10.7t - 240.9 & 17 \le t \le 22 \\ -5.5 & t \ge 22 \end{cases}$$

Also find the antiderivative of $v(t)$, we have the position function $s(t)$:

$$s(t) = \begin{cases} 3t^3 + D_1 & 0 \le t \le 3 \\ -5t^2 + 111t + D_2 & 3 \le t \le 17 \\ 5.35t^2 - 240.9t + D_3 & 17 \le t \le 22 \\ -5.5t + D_4 & t \ge 22 \end{cases}$$

Then $s(0) = 0 \Rightarrow D_1 = 0$, $s(3) = 81$; $s(3) = 81 \Rightarrow D_2 = -207$, $s(17) = 235$; $s(17) = 235 \Rightarrow D_3 = 2784.15$, $s(22) = 73.75$; $s(22) = 73.75 \Rightarrow D_4 = 194.75$. So,

$$s(t) = \begin{cases} 3t^3 & 0 \le t \le 3 \\ -5t^2 + 111t - 207 & 3 \le t \le 17 \\ 5.35t^2 - 240.9t + 2784.15 & 17 \le t \le 22 \\ -5.5t + 194.75 & t \ge 22 \end{cases}$$

The graph of $v(t)$ and $s(t)$:

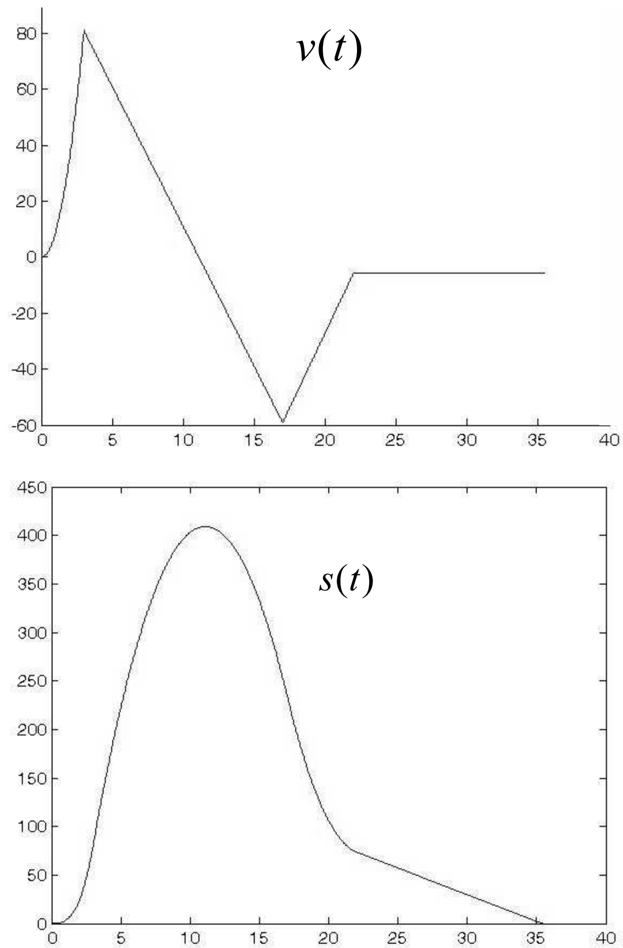

Figure 3: 4.9 Ex.76

(b) The velocity is zero when the rocket reaches its maximum height, so we let $v(t) = 0$, get $t = 0$ or 11.1. The maximum height is $s(11) = 409.05$(m).

(c) The position is zero when the rocket lands, so we let $s(t) = 0$, get $t = 0$ or 35.41. So $t = 35.41$(sec).