Section 4.8

3. Since x_1=3, and y=5x-4 is tangent to function f(x) at x=3, we must find the intersection between y=5x-4 and y=0. Clearly, we can get the $x_2=\frac{4}{5}$.

12.

we can consider function $f(x)=x^{100}-100$

$\Rightarrow \frac{df(x)}{dx}=100x^{99}$, so we have $x_{n+1}=x_n-\frac{x_n^{100}-100}{100x_n^{99}}$

We must find approximations until they agree to eight decimal places. Start for $x_1=2 \Rightarrow x_2\approx$1.980000000000000

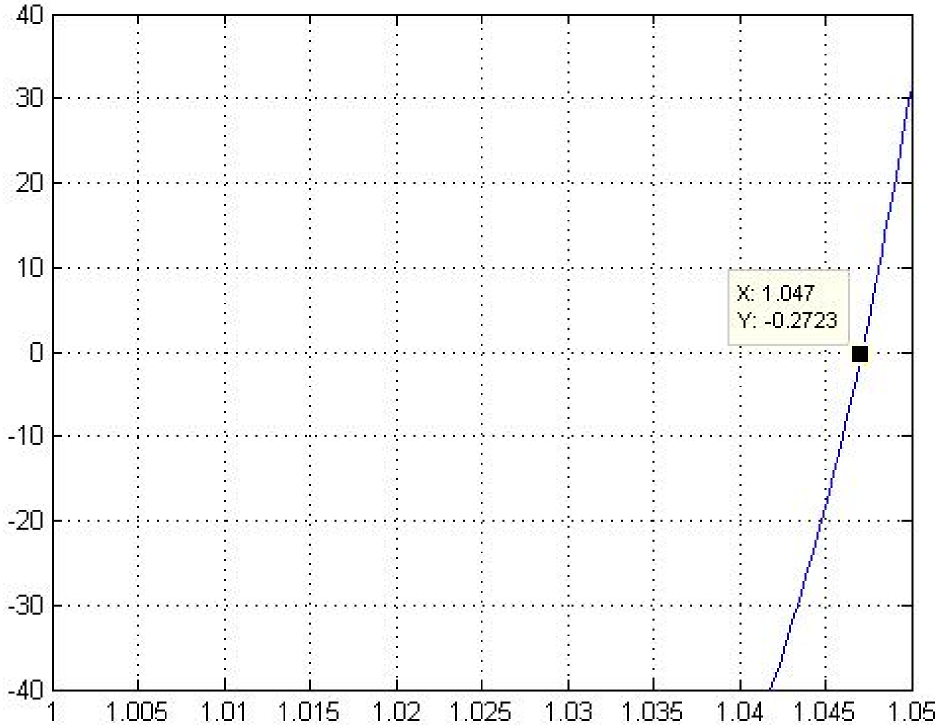
Use matlab we observe the $x_{n1}\approx$1.047131037167433

$x_{n2}\approx$1.047128548343760

$x_{n3}\approx$1.047128548050900 for some n1 n2 n3.

We can take $\sqrt[100]{100}\approx$1.047128548050900 to agree to eight decimal places.

(the graph can help us to take a good initial value for approaching root.)



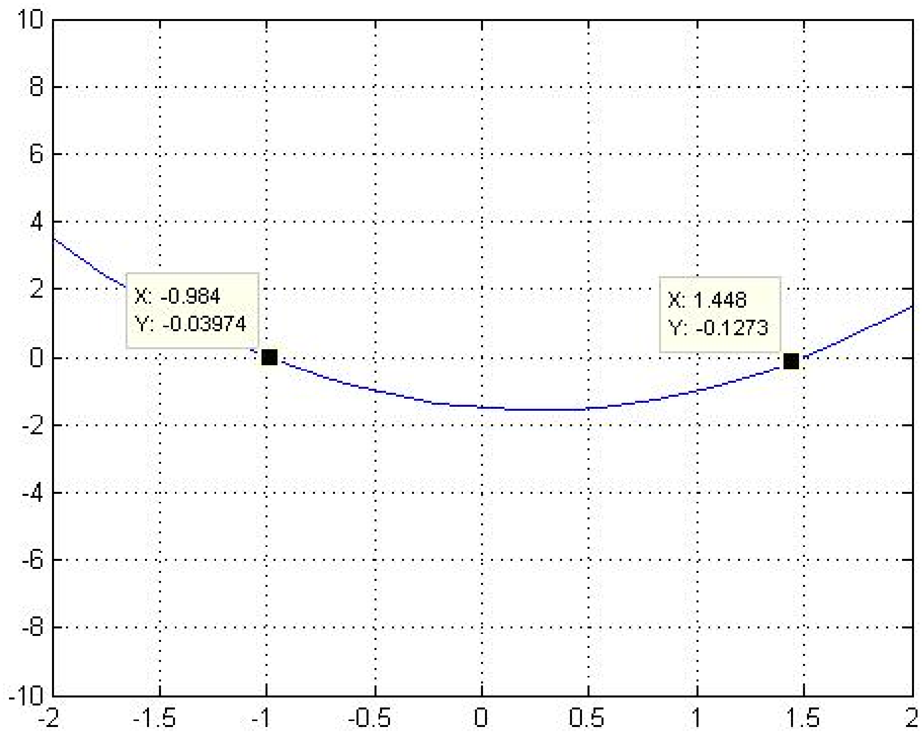
22. By the same way, we consider $f(x)=x^2-\sqrt{x+3}$

$\Rightarrow \frac{df(x)}{dx}=2x-\frac{1}{2\sqrt{x+3}}$, so

$$x_{n+1}=x_n-\frac{x_n^2-\sqrt{x_n+3}}{2x_n-1\Big/(2\sqrt{x_n+3})}$$

Use matlab we have (i) $x_1=-1.2$, $x_2\approx-1.164526$ $x_3\approx-1.164035$. (ii) $x_1=1.5$, $x_2\approx1.453449$, $x_3\approx1.452627$.

To six decimal places the roots of equation are -1.164035 and 1.452627.



30.

(a) since, $f(x)=\frac{1}{x}-a \Rightarrow \frac{df(x)}{dx}=-\frac{1}{x^2}$, we have

$$x_{n+1}=x_n-\frac{\frac{1}{x_n}-a}{\frac{-1}{x_n^2}}=2x_n-ax_n^2.$$

(b) use part (a) with a=1.6984 and $x_1=0.5$, we get

$x_2=0.5754$, $x_3\approx 0.588485$, and $x_4\approx 0.588789$.

So $\frac{1}{1.6984}\approx 0.588789$.

31. Since $f(x)=x^3-3x+6 \Rightarrow \frac{df(x)}{dx}=3x^2-3$

Consider $x_1=1$, then$\left(\frac{df(x)}{dx}\Big/_{x=1}\right)=0$ and tangent line used for approximating x_2 is horizontal. Attempting to find x_2 results in trying to divide by zero.

32. Consider $f(x)=x^3-x-1 \Rightarrow \frac{df(x)}{dx}=3x^2-1$

So, we have $x_{n+1}=x_n-\frac{x_n^3-x_n-1}{3x_n^2-1}$.

(a) $x_1=1$ $x_2=1.5$ $x_3 \approx 1.347826, x_4 \approx 1.325200$

$x_5 \approx 1.324718 \approx x_6$.

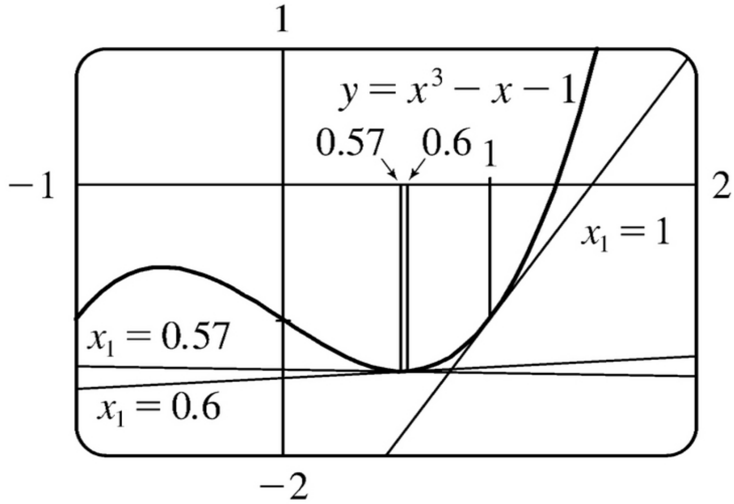
(b) $x_1=0.6$, $x_2=17.9$, $x_3 \approx 11.946802, x_4 \approx 7.985520$

$\cdots x_{11} \approx 1.324913$, $x_{12} \approx 1.324718 \approx x_{13}$.

(c) Since $x_1=0.57$, $x_2 \approx -54.165455$, $x_3 \approx -36.114293$

$\cdots$ $x_{35} \approx 1.324719$, $x_{36} \approx 1.324718 \approx x_{37}$.

(d)



from the figure, we see that the tangent line corresponding to x_1=1 results in a sequence of approximations that converges quite quickly ($x_5 \approx x_6$). the tangent line corresponding to x_1=0.6 is close to being horizontal, so x_2 is quite far from the root. But the sequence still converges just alittle more slowly ($x_{12} \approx x_{13}$) Lastly, the tangent line corresponding to x_1=0.57 is very nearly horizontal, x_2 is farther away from the root, and the sequence takes more iterations to converge ($x_{36} \approx x_{37}$).

41.

consider the case , A=18000, R=375, and n=5*12=60.

So the formula is $A=\frac{R}{i}[1-(1+i)^{-n}]$ becomes

$18000=\frac{375}{x}[1-(1+x)^{-60}] \Leftrightarrow 48x = 1-(1+x)^{-60}$

$\Leftrightarrow 48x(1+x)^{60} - (1+x)^{60}+1 = 0$. Let it be called f(x).

We have

$\frac{df(x)}{dx} = 48x(60)(1+x)^{59} + 48(1+x)^{59} - 60(1+x)^{59}$

$= 12(1+x)^{59}(244x-1)$,

$x_{n+1} = x_n - \frac{48x_n(1+x_n)^{60}-(1+x_n)^{60}+1}{12(1+x_n)^{59}(244x_n-1)}$.

An interest rate of 1% per month seems like a reasonable estimate for x=i. So let $x_1=1\% = 0.01$, and we get

$X_2 \approx 0.0082202$, $x_3 \approx 0.0076802$, $x_4 \approx 0.0076291$,

$x_5 \approx 0.0076286 \approx x_6$.

Therefore, the dealer is charging a monthly interest rate of 0.76286%.