4-6 Graphing with Calculus and Calculators

2. $f(x) = 8x^5 + 45x^4 + 80x^3 + 90x^2 + 200x, f(0) = 0.$
$f'(x) = 40x^4 + 180x^3 + 240x^2 + 180x + 200 = 20(x^2+1)(x+2)(2x+5).$
$f''(x) = 160x^3 + 540x^2 + 480x + 180, f''(-2.27284) \approx 0.$
Intervals of increase: $(-\infty, -\frac{5}{2}), (-2, +\infty)$.
Interval of decrease: $(-\frac{5}{2}, -2)$.
concave upward on $(-2.27284, +\infty)$.
concave downward on $(-\infty, -2.27284)$.

4. $f(x) = \dfrac{x^2-1}{40x^3+x+1}, f(1) = 0 = f(-1).$
$\Rightarrow$vertical asymptote:$x = -0.26337$.
Since $\lim_{x\to+\infty} f(x) = 0 = \lim_{x\to-\infty} f(x) \Rightarrow$ horizontal asymptote:$y = 0$
$f'(x) = \dfrac{2x(40x^3+x+1) - (x^2-1)(120x^2+1)}{(40x^3+x+1)^2} = \dfrac{-40x^4+121x^2+2x+1}{(40x^3+x+1)^2}$
$f'(1.75134) \approx 0, f'(-1.72460) \approx 0$
Interval of increase:$(-1.72460, 1.75134)$
Interval of decrease:$(-\infty, -1.72460), (1.75134, +\infty)$

12. $f(x) = xe^{\frac{1}{x}}$
$\lim_{x\to 0+} xe^{\frac{1}{x}} = \lim_{x\to 0+} \dfrac{e^{\frac{1}{x}}}{\frac{1}{x}} = \lim_{t\to+\infty} \dfrac{e^t}{t} = \lim_{t\to+\infty} \dfrac{e^t}{1} = +\infty.$
$\lim_{x\to 0-} xe^{\frac{1}{x}} = \lim_{x\to 0-} \dfrac{e^{\frac{1}{x}}}{\frac{1}{x}} = \lim_{t\to-\infty} \dfrac{e^t}{t} = \lim_{t\to-\infty} \dfrac{e^t}{1} = 0.$
$f'(x) = e^{\frac{1}{x}} + xe^{\frac{1}{x}}(-1)x^{-2} = e^{\frac{1}{x}}(1 - \dfrac{1}{x}).$
$x < 0 \Rightarrow f'(x) > 0 \Rightarrow f$ is increasing.
$0 < x < 1 \Rightarrow f'(x) < 0 \Rightarrow f$ is decreasing.
$x > 1 \Rightarrow f'(x) > 0 \Rightarrow f$ is increasing.
$f''(x) = e^{\frac{1}{x}}(-x^{-2})(1 - \dfrac{1}{x}) + e^{\frac{1}{x}}x^{-2} = \dfrac{1}{x^3}e^{\frac{1}{x}}.$
$x < 0 \Rightarrow f''(x) < 0 \Rightarrow$ concave downward.
$x > 0 \Rightarrow f''(x) > 0 \Rightarrow$ concave upward.
$f'(1) = 0, f''(1) > 0 \Rightarrow f(1) = e$ is a local minimum.

14. Omitted.

30. $f(x) = \ln(x^2 + c), f'(x) = \dfrac{2x}{x^2+c}, f''(x) = \dfrac{-2x^2+2c}{(x^2+c)^2}$
$c > 0 \Rightarrow x^2 + c > 0$ is OK.
$c < 0 \Rightarrow x^2 > -c \Rightarrow |x| > \sqrt{-c}.$
If $x > 0$, then $f'(x) > 0 \Rightarrow f$ is increasing, no matter what c is.
If $x < 0$, then $f'(x) < 0 \Rightarrow f$ is decreasing, no matter what c is.

$-2x^2 + 2c = 0 \Rightarrow 2x^2 = 2c \Rightarrow x = \sqrt{c}, -\sqrt{c}$.
So If $c > 0$, there are two inflection points at $x = \sqrt{c}$ and $x = -\sqrt{c}$.
f is concave upward if $|x| < \sqrt{c}$ and concave downward if $|x| > \sqrt{c}$.
If $c = 0$, there is no inflection point since f"(x) does not change sign on both sides of $x = 0$.
If $c < 0$, there is no inflection point.

35. $f'(x) = e^{-cx} + x(-c)e^{-cx} = e^{-cx}(1 - cx)$,
$f''(x) = -ce^{-cx}(1-cx) - ce^{-cx} = -2ce^{-cx} + c^2xe^{-cx} = e^{-cx}(-2c + c^2x)$.
If $c \neq 0$, then $1 - cx = 0 \Rightarrow x = \frac{1}{c}$.
So if $x > \frac{1}{c}$, then $f'(x) < 0 \Rightarrow f$ is decreasing.
If $x < \frac{1}{c}$, then $f'(x) > 0 \Rightarrow f$ is increasing.
$-2c + c^2x = c(cx - 2) = 0 \Rightarrow c = 0$ or $cx = 2 \Rightarrow c = 0$ or $x = \frac{2}{c}$.
So if $c > 0, x > \frac{2}{c}$ or $c < 0, x > \frac{2}{c}$, then $f''(x) > 0$.
$\Rightarrow$ if $x > \frac{2}{c}$, then f is concave upward.
If $c > 0, x < \frac{2}{c}$ or $c < 0, x < \frac{2}{c}$, then $f''(x) < 0$.
$\Rightarrow$ if $x < \frac{2}{c}$, then f is concave downward.
$f(\frac{1}{c}) = \frac{1}{ce}$ is the maximum value, and $(\frac{2}{c}, \frac{2}{ce^2})$ is the inflection point.
If $c = 0$, then $f(x) = x \Rightarrow$ no local extreme value, no inflection point, and both concave upward and concave downward.