Section 4.5

3 $y = f(x) = 2 - 15x + 9x^2 - x^3 = -(x-2)(x^2 - 7x + 1)$

A. $D = \mathbb{R}$

B. y-intercept: $f(0) = 2$;
x-intercept: $f(x) = 0 \Rightarrow x = 2 \quad or \quad x = \frac{7 \pm \sqrt{45}}{2}$

C. No symmetry

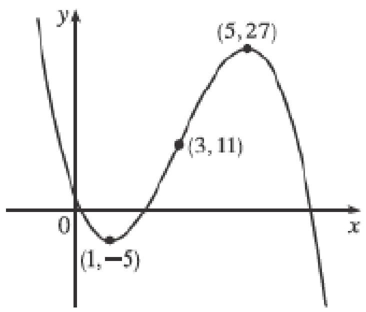
D. No asymptote

E. $f'(x) = -15 + 18x - 3x^2 = -3(x-1)(x-5)$
So $f(x)$ is increasing on $(1, 5)$, and decreasing on $(-\infty, 1)$ and $(5, \infty)$.

F. Local maximum :$f(5) = 27$. Local minimum : $f(1) = -5$.

G. $f''(x) = 18 - 6x$
So $f(x)$ is concave up on $(-\infty, 3)$, and concave down on $(3, \infty)$.
Inflection point : $(3, 10)$.

H.



26 $y = f(x) = \frac{x}{\sqrt{x^2-1}}$

A. $D = (-\infty, -1) \cup (1, \infty)$

B. No instercepts.

C. $f(-x) = -f(x)$, symmetric about the origin.

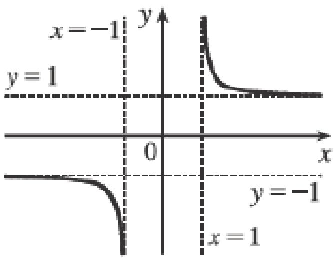
D. $\lim_{x\to\infty} f(x) = 1$, $\lim_{x\to-\infty} f(x) = -1$, so $y = \pm 1$ are horizontal asymptotes.
And $x = \pm 1$ are vertical asymptotes.

E. $f'(x) = \frac{-1}{(x^2-1)^{3/2}} < 0$, So $f(x)$ is decreasing on D.

F. No.

G. $f''(x) = \frac{3x}{(x^2-1)^{5/2}}$
So $f(x)$ is concave up on $(1, \infty)$, and concave down on $(-\infty, -1)$.
No inflection point.

H.



47 $y = f(x) = \ln(\sin(x))$

A. $D = \{x \in \mathbb{R} | \sin(x) > 0\} = \{\cup(2n\pi, (2n+1)\pi)\}$

B. No y-intercept.
x-intercepts : $x = 2n\pi + \pi/2$.

C. $f(x)$ is periodic with period 2π.

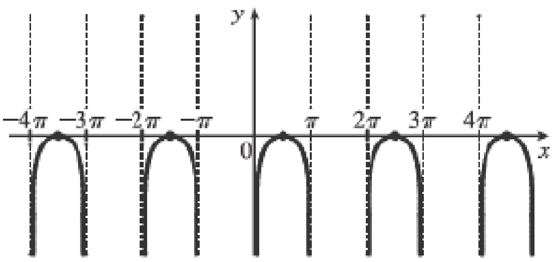
D. $x = n\pi$ are vertical asymptotes.

E. $f'(x) = \cot(x)$, So $f(x)$ is increasing on $(2n\pi, 2n\pi + \pi/2)$, and decreasing on $(-\infty, 1)$ and $(2n\pi + \pi/2, (2n+1)\pi)$.

F. Local maximum at $f(2n\pi + \pi/2) = 0$. No local minimum.

G. $f''(x) = -\csc^2(x) < 0$
So $f(x)$ is concave down on D.
No inflection point.

H.



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$D = (0, 2)$

$F(1) = 0 \Rightarrow x - intercepts : x = 1$

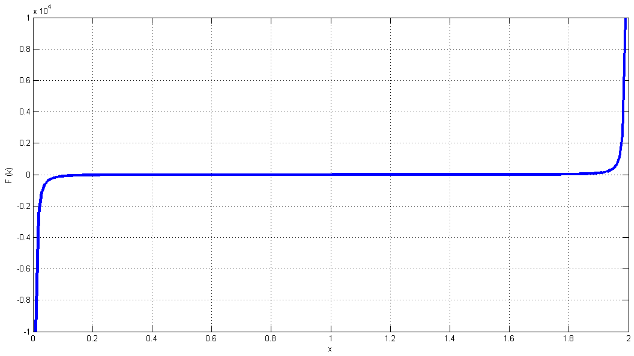
$F'(x) = \frac{2k}{x^3} - \frac{2k}{(x-2)^3} = -4k\frac{3x^2-6x+4}{x^3(x-2)^3} > 0$

$F''(x) = \frac{-6k}{x^4} + \frac{6k}{(x-2)^4} = 48k\frac{x^3-3x^2+4x-2}{x^4(x-2)^4} = 48k\frac{(x-1)(x^2-2x+2)}{x^4(x-2)^4}$

So $F(x)$ is concave up on $(0, 1)$, and concave down on $(1, 2)$

$\lim_{x\to 0} F(x) = -\infty, \lim_{x\to 2} F(x) = \infty$

So $x = 0$ and $x = 2$are vertical asymptotes.



60 $\lim_{x\to\infty} y/x = 5, m = 5$

$y - (5x + b) = \frac{6x^2+x-10}{x^3-x^2+2} - b \longrightarrow 0 \quad as \quad x \longrightarrow \infty$

So $b = 0$, the slant asymptote is $y = 5x$.

67 $\lim_{x\to\infty}(y - x) = \lim_{x\to\infty} - \tan^{-1}(x) = -\pi/2$

$\lim_{x\to-\infty}(y - x) = \lim_{x\to\infty} - \tan^{-1}(x) = \pi/2$

So $y = x - \pi/2$ and $y = x + \pi/2$ are two slant asymptotes.

70 $\lim_{x\to\pm\infty} [f(x) - x^2] = \lim_{x\to\pm\infty} \frac{1}{x} = 0$