11. (a) $f(x) = x^4 - 2x^2 + 3 \Rightarrow f'(x) = 4x^3 - 4x = 4x(x^2-1) = 4x(x+1)(x-1)$.

Interval	$x+1$	x	$x-1$	$f'(x)$	f
$x < -1$	−	−	−	−	decreasing on $(-\infty, -1)$
$-1 < x < 0$	+	−	−	+	increasing on $(-1, 0)$
$0 < x < 1$	+	+	−	−	decreasing on $(0, 1)$
$x > 1$	+	+	+	+	increasing on $(1, \infty)$

(b) f changes from increasing to decreasing at $x = 0$ and from decreasing to increasing at $x = -1$ and $x = 1$. Thus, $f(0) = 3$ is a local maximum value and $f(\pm 1) = 2$ are local minimum values.

(c) $f''(x) = 12x^2 - 4 = 12(x^2 - \frac{1}{3}) = 12((x + \frac{1}{\sqrt{3}})(x - \frac{1}{\sqrt{3}}))$.

$f''(x) > 0 \Leftrightarrow x < -\frac{1}{\sqrt{3}}$ or $x > \frac{1}{\sqrt{3}}$ and $f''(x) < 0 \Leftrightarrow -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$.

Thus, f is concave upward on $(-\infty, -\frac{\sqrt{3}}{3})$ and $(\frac{\sqrt{3}}{3}, \infty)$ and concave downward on $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$. There are inflection points at $(\pm\frac{\sqrt{3}}{3}, \frac{22}{9})$.

12. (a) $f(x) = \frac{x^2}{x^2+3} \Rightarrow f'(x) = \frac{(x^2+3)(2x) - x^2(2x)}{(x^2+3)^2} = \frac{6x}{(x^2+3)^2}$. The denominator is positive so the sign of $f'(x)$ is determined by the sign of x. Thus, $f'(x) > 0 \Leftrightarrow x > 0$ and $f'(x) < 0 \Leftrightarrow x < 0$. So f is increasing on $(0, \infty)$ and f is decreasing on $(-\infty, 0)$.

(b) f changes from decreasing to increasing at $x = 0$. Thus, $f(0) = 0$ is a local minimum value.

(c) $f''(x) = \frac{(x^2+3)^2(6) - 6x \cdot 2(x^2+3)(2x)}{[(x^2+3)^2]^2} = \frac{6(x^2+3)[x^2+3-4x^2]}{(x^2+3)^4}$

$= \frac{6(3-3x^2)}{(x^2+3)^3} = \frac{-18(x+1)(x-1)}{(x^2+3)^3}$.

$f''(x) > 0 \Leftrightarrow -1 < x < 1$ and $f''(x) < 0 \Leftrightarrow x < -1$ or $x > 1$. Thus, f is concave upward on $(-1, 1)$ and concave downward on $(-\infty, -1)$ and $(1, \infty)$. There are inflection points at $(\pm 1, \frac{1}{4})$.

16. (a) $f(x) = x^2 \ln x \Rightarrow f'(x) = x^2(1/x) + (\ln x)(2x) = x + 2x \ln x = x(1 + 2\ln x)$. The domain of f is $(0, \infty)$, so the sign of f' is determined solely by the factor $1 + 2\ln x$. $f'(x) > 0 \Leftrightarrow \ln x > -\frac{1}{2} \Leftrightarrow x > e^{\frac{-1}{2}} [\approx 0.61]$ and $f'(x) < 0 \Leftrightarrow 0 < x < e^{\frac{-1}{2}}$. So f is increasing on $(e^{\frac{-1}{2}}, \infty)$ and f is decreasing on $(0, e^{\frac{-1}{2}})$.

(b) f changes from decreasing to increasing at $x = e^{\frac{-1}{2}}$. Thus, $f(e^{\frac{-1}{2}}) = (e^{\frac{-1}{2}})^2 \ln(e^{\frac{-1}{2}}) = e^{-1}(\frac{-1}{2}) = -\frac{1}{2e}$ [≈ -0.18] is a local minimum value.

(c) $f'(x) = x(1+2\ln x) \Rightarrow f''(x) = x(2/x)+(1+2\ln x)\cdot 1 = 2+1+2\ln x = 3+2\ln x$. $f''(x) > 0 \Leftrightarrow 3+2\ln x > 0 \Leftrightarrow \ln x > -3/2 \Leftrightarrow x > e^{-3/2}$ [≈ 0.22]. Thus, f is concave upward on $(e^{-3/2}, \infty)$ and f is concave downward on $(0, e^{-3/2})$. $f(e^{-3/2}) = (e^{-3/2})^2 \ln e^{-3/2} = e^{-3}(-3/2) = \frac{-3}{2e^3}$ [≈ -0.07]. There is a point of inflection at $(e^{-3/2}, f(e^{-3/2})) = (e^{-3/2}, \frac{-3}{2e^3})$.

20. $f(x) = \frac{x}{x^2+4} \Rightarrow f'(x) = \frac{(x^2+4)\cdot 1 - x(2x)}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2} = \frac{(2+x)(2-x)}{(x^2+4)^2}$.

First Derivative Test: $f'(x) > 0 \Rightarrow -2 < x < 2$ and $f'(x) < 0 \Rightarrow x > 2$ or $x < -2$. Since f' changes from positive to negative at $x = 2, f(2) = \frac{1}{4}$ is a local maximum value; and since f' changes from negative to positive at $x = -2$, $f(-2) = -\frac{1}{4}$ is a local minimum value.

Second Derivative Test: $f''(x) = \frac{(x^2+4)^2(-2x)-(4-x^2)\cdot 2(x^2+4)(2x)}{[(x^2+4)^2]^2}$

$= \frac{-2x(x^2+4)[(x^2+4)+2(4-x^2)]}{(x^2+4)^4} = \frac{-2x(12-x^2)}{(x^2+4)^3}$.

$f'(x) = 0 \Leftrightarrow x = \pm 2$.

$f''(-2) = \frac{1}{16} > 0 \Rightarrow f(-2) = -\frac{1}{4}$ is a local minimum value.

$f''(2) = -\frac{1}{16} < 0 \Rightarrow f(2) = \frac{1}{4}$ is a local maximum value.

Preference: Since calclating the second derivative is fairly difficult, the First Derivative Test is easier to use for this function.

21. $f(x) = x + \sqrt{1-x} \Rightarrow f'(x) = 1 + \frac{1}{2}(1-x)^{-1/2}(-1) = 1 - \frac{1}{2\sqrt{1-x}}$. Note that f is defined for $1 - x \geq 0$; that is, for $x \leq 1$. $f'(x) = 0 \Rightarrow 2\sqrt{1-x} = 1 \Rightarrow \sqrt{1-x} = \frac{1}{2} \Rightarrow 1 - x = \frac{1}{4} \Rightarrow x = \frac{3}{4}$. f' does not exist at $x = 1$, but we can't have a local maximum of minimum at an endpoint.

First Derivative Test: $f'(x) > 0 \Rightarrow x < \frac{3}{4}$ and $f'(x) < 0 \Rightarrow \frac{3}{4} < x < 1$. Since f' changes from positive to negative at $x = \frac{3}{4}$, $f(\frac{3}{4}) = \frac{5}{4}$ is a local maximum value.

Second Derivative Test: $f''(x) = -\frac{1}{2}(-\frac{1}{2})(1-x)^{-3/2}(-1) = -\frac{1}{4(\sqrt{1-x})^3}$.
$f''(\frac{3}{4}) = -2 < 0 \Rightarrow f(\frac{3}{4}) = \frac{5}{4}$ is a local maximum value.

Preference: The First Derivative Test may be slightly easier to apply in this case.

22. (a) $f(x) = x^4(x-1)^3 \Rightarrow f'(x) = x^4 \cdot 3(x-1)^2 + (x-1)^3 \cdot 4x^3$
$= x^3(x-1)^2[3x + 4(x-1)] = x^3(x-1)^2(7x-4)$.

The critical numbers are 0, 1, and $\frac{4}{7}$.

(b) $f''(x) = 3x^2(x-1)^2(7x-4) + x^3 \cdot 2(x-1)(7x-4) + x^3(x-1)^2 \cdot 7$
$= x^2(x-1)[3(x-1)(7x-4) + 2x(7x-4) + 7x(x-1)]$.

Now $f''(0) = f''(1) = 0$, so the Second Derivative Test gives no information for $x = 0$ or $x = 1$.

$f''(\frac{4}{7}) = (\frac{4}{7})^2(\frac{4}{7}-1)[0+0+7(\frac{4}{7})(\frac{4}{7}-1)] = (\frac{4}{7})^2(-\frac{3}{7})(4)(-\frac{3}{7}) > 0$, so there is a local minimum at $x = \frac{4}{7}$.

(c) f' is positive on $(-\infty, 0)$, negative on $(0, \frac{4}{7})$, positive on $(\frac{4}{7}, 1)$, and positive on $(1, \infty)$. So f has a local maximum at $x = 0$, a local minimum at $x = \frac{4}{7}$, and no local maximum or minimum at $x = 1$.

41. (a) $C(x) = x^{1/3}(x+4) = x^{4/3} + 4x^{1/3}$

$\Rightarrow C'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3} = \frac{4}{3}x^{-2/3}(x+1) = \frac{4(x+1)}{3\sqrt[3]{x^2}}$.

$C'(x) > 0$ if $-1 < x < 0$ or $x > 0$ and $C'(x) < 0$ for $x < -1$,

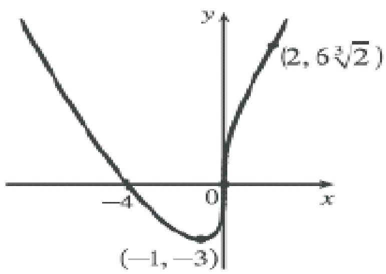
so C is increasing on $(-1, \infty)$ and C is decreasing on $(-\infty, -1)$.

(b) $C(-1) = -3$ is a local minimum value.

(c) $C''(x) = \frac{4}{9}x^{-2/3} - \frac{8}{9}x^{-5/3} = \frac{4}{9}x^{-5/3}(x-2) = \frac{4(x-2)}{9\sqrt[3]{x^5}}$.

$C''(x) < 0$ for $0 < x < 2$ and $C''(x) > 0$ for $x < 0$ and $x > 2$, so C is concave downward on $(0, 2)$ and concave upward on $(-\infty, 0)$ and $(2, \infty)$. There are inflection points at $(0, 0)$ and $(2, 6\sqrt[3]{2}) \approx (2, 7.56)$.

(d)



76. (a) Let $f(x) = e^x - 1 - x \Rightarrow f'(x) = e^x - 1$ for $x \geq 0$.

Because $f'(x) \geq 0$ for $x \geq 0 \Rightarrow f$ is increasing on $(0, \infty)$

$\Rightarrow f(x) \geq f(0) = 0$ for $x \geq 0$.

$\Rightarrow e^x \geq 1 + x$ for $x \geq 0$.

(b) Let $g(x) = e^x - 1 - x - \frac{1}{2}x^2 \Rightarrow g'(x) = e^x - 1 - x$ for $x \geq 0$.

By (a), $g'(x) \geq 0$ for $x \geq 0 \Rightarrow g$ is increasing on $(0, \infty)$

$\Rightarrow g(x) \geq g(0) = 0$ for $x \geq 0$.

$\Rightarrow e^x \geq 1 + x + \frac{1}{2}x^2$ for $x \geq 0$.

(c) As $n = 1$, by (a), we know $e^x \geq 1 + x$ for $x \geq 0$.

As $n = k$, $e^x \geq 1 + x + \frac{1}{2}x^2 + ... + \frac{1}{k!}x^k$ is holded for $x \geq 0$.

As $n = k+1$, Let $h(x) = e^x - (1 + x + \frac{1}{2}x^2 + ... + \frac{1}{(k+1)!}x^{(k+1)})$

$h'(x) = e^x - (1 + x + \frac{1}{2}x^2 + ... + \frac{1}{k!}x^k) \geq 0$ for $x \geq 0$.

i.e. h is increasing for $x \geq 0$,

then $h(x) \geq h(0) = 0$ for $x \geq 0$

$\Rightarrow e^x \geq 1 + x + \frac{1}{2}x^2 + ... + \frac{1}{(k+1)!}x^{(k+1)}$ for $x \geq 0$.

By induction, we know $e^x \geq 1 + x + \frac{1}{2}x^2 + ... + \frac{1}{n!}x^n$ for $x \geq 0$ any positive integer n.