12.

(a) Given the rate of decrease of the surface area is 1 cm^2/min. If we let t be time(in minutes) and S be the surface area(in cm^2), then we are given that $dS/dt = -1\ cm^2/min$.

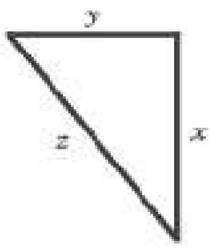
(b) Unknow: the rate of decrease of the diameter when the diameter is 10 cm. If we let x be the diameter, then we want to find dx/dt when $x = 10\ cm$.

(c) If the radius is r and the diameter $x = 2r$, then $r = \frac{1}{2}x$ and

$$S = 4\pi r^2 = 4\pi(\frac{1}{2}x)^2 = \pi x^2 \Rightarrow \frac{dS}{dt} = \frac{dS}{dx}\frac{dx}{dt} = 2\pi x\frac{dx}{dt}.$$

(d)$-1 = \frac{dS}{dt} = 2\pi x\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -\frac{1}{2\pi x}$. When $x = 10$, $\frac{dx}{dt} = -\frac{1}{20\pi}$. So the rate of decrease is $\frac{1}{20\pi}\ cm/min$.

15.



We are given that $\frac{dx}{dt} = 60\ mi/h$ and $\frac{dy}{dt} = 25\ mi/h$. $z^2 = x^2 + y^2 \Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \Rightarrow z\frac{dz}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt} \Rightarrow \frac{dz}{dt} = \frac{1}{z}(2x\frac{dx}{dt} + 2y\frac{dy}{dt})$. After 2 hours, $x = 2(60) = 120$ and $y = 2(25) = 50 \Rightarrow z = \sqrt{120^2 + 50^2} = 130$, so $\frac{dz}{dt} = \frac{1}{z}(x\frac{dx}{dt} + y\frac{dy}{dt}) = \frac{120(60)+50(25)}{130} = 65\ mi/h$.

19.
$A = \frac{1}{2}bh$, where b is the base and h is the altitude. We are given that $\frac{dh}{dt} = 1\ cm/min$ and $\frac{dA}{dt} = 2\ cm^2/min$. Using the product rule, we have $\frac{dA}{dt} = \frac{1}{2}(b\frac{dh}{dt} + h\frac{db}{dt})$. When $h = 10$ and $A = 100$, we have $100 = \frac{1}{2}b(10) \Rightarrow \frac{1}{2}b = 10 \Rightarrow b = 20$, so $2 = \frac{1}{2}(20 \cdot 1 + 10\frac{db}{dt}) \Rightarrow 4 = 20 + 10\frac{db}{dt} \Rightarrow \frac{db}{dt} = \frac{4-20}{10} = -1.6\ cm/min.$

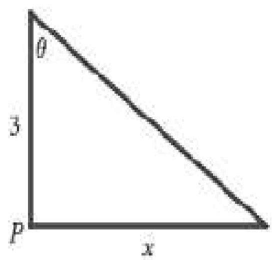
22.
Let D denote the distance from the origin $(0,0)$ to the point on the curve $y = \sqrt{x}$.

$$D = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (\sqrt{x})^2} = \sqrt{x^2 + x}$$

$$\Rightarrow \frac{dD}{dt} = \frac{1}{2}(x^2 + x)^{1\frac{1}{2}}(2x+1)\frac{dx}{dt} = \frac{2x+1}{2\sqrt{x^2+x}}\frac{dx}{dt}.$$

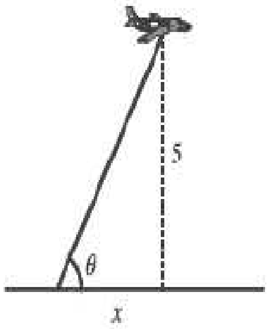
With $\frac{dx}{dt} = 3$ when $x = 4$, $\frac{dD}{dt} = \frac{9}{2\sqrt{20}}(3) = \frac{27}{4\sqrt{5}} \approx 3.02\ cm/s.$

38.



We are given that $\frac{d\theta}{dt} = 4(2\pi) = 8\pi\ rad/min. x = 3tan\theta \Rightarrow \frac{dx}{dt} = 3sec^2\theta\frac{d\theta}{dt}$. When $x = 1$, $tan\theta = \frac{1}{3}$, so $sec^2\theta = 1 + (\frac{1}{3})^2 = \frac{10}{9}$ and $\frac{dx}{dt} = 3(\frac{10}{9})(8\pi) = \frac{80}{3}\pi \approx 83.8\ km/min.$

39.



$cot\theta = \frac{x}{5} \Rightarrow -csc^2\theta \ \frac{d\theta}{dt} = \frac{1}{5}\frac{dx}{dt} \Rightarrow -(csc\frac{\pi}{3})^2(-\frac{\pi}{6}) = \frac{1}{5}\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{5\pi}{6}(\frac{2}{\sqrt{3}})^2 = \frac{10}{9}\pi \ km/min.$