Section 3.8

3. (a) $y = 100 \times (4.2)^t$.

   (b) $y = 100 \times (4.2)^3 \approx 7409$.

   (c) $\dfrac{dy}{dt} = \ln 4.2 \times 100 \times (4.2)^t$.

   (d)

$$\begin{aligned} & 100 \times (4.2)^t = 10000 \\ \Rightarrow & (4.2)^t = 100 \\ \Rightarrow & t \ln 4.2 = \ln 100 \\ \Rightarrow & t = \frac{\ln 100}{\ln 4.2} \approx 3.2 \end{aligned}$$

5. (a) The model is $y = 790 \cdot \left(\dfrac{980}{790}\right)^{(t-1750)/50}$ and

$$y(1900) = 790 \cdot \left(\frac{980}{790}\right)^{(1900-1750)/50} \approx 1508$$

$$y(1950) = 790 \cdot \left(\frac{980}{790}\right)^{(1950-1750)/50} \approx 1870$$

   (b) The model is $y = 1260 \cdot \left(\dfrac{1650}{1260}\right)^{(t-1850)/50}$ and

$$y(1950) = 1260 \cdot \left(\frac{1650}{1260}\right)^{(1950-1850)/50} \approx 2161$$

   (c) The model is $y = 1650 \cdot \left(\dfrac{2560}{1650}\right)^{(t-1900)/50}$ and

$$y(2000) = 1650 \cdot \left(\frac{2560}{1650}\right)^{(2000-1900)/50} \approx 3972$$

   Note the population growthing rate is not the proportional the the population size, thus the exponential is not appropriate.

9. (a) $M(t) = 100 \left(\dfrac{1}{2}\right)^{t/30}$.

   (b) $M(100) = 100 \left(\dfrac{1}{2}\right)^{100/30} \approx 9.9213$.

   (c)

$$\begin{aligned} & 100 \left(\frac{1}{2}\right)^{t/30} = 1 \\ \Rightarrow & \left(\frac{1}{2}\right)^{t/30} = \frac{1}{100} \\ \Rightarrow & \frac{t}{30} \ln \frac{1}{2} = \ln \frac{1}{100} \\ \Rightarrow & t = 30 \cdot \frac{\ln(1/100)}{\ln(1/2)} \approx 199.3 \end{aligned}$$

11.

$$\begin{aligned}
&\left(\frac{1}{2}\right)^{t/5730} = 74\% \\
\Rightarrow &\frac{t}{5730}\ln\frac{1}{2} = \ln\frac{74}{100} \\
\Rightarrow &t = 5730 \cdot \frac{\ln(74/100)}{\ln(1/2)} \approx 2489.1
\end{aligned}$$

12. $y(t) = y(0)e^{2t} = 5e^{2t}$.

20. (a)

$$\begin{aligned}
&e^{0.06t} = 2 \\
\Rightarrow &0.06t = \ln 2 \\
\Rightarrow &t = \frac{1}{0.06} \cdot \ln 2 \approx 11.55
\end{aligned}$$

(b)

$$\begin{aligned}
&(1+x)^{\frac{\ln 2}{0.06}} = 2 \\
\Rightarrow &\ln(1+x) = 0.06 \\
\Rightarrow &1 + x \approx 1.0618 \\
\Rightarrow &x \approx 6.18\%
\end{aligned}$$