Section 3.7(Solutions)

19. The current is $\frac{dQ}{dt} = 3t^2 - 4t + 6$.
(a) $\frac{dQ}{dt}\Big|_{t=0.5} = 4.75$
(b) $\frac{dQ}{dt}\Big|_{t=1} = 5$
$\frac{dQ}{dt} = 3(t - \frac{2}{3})^2 + \frac{14}{3} \Rightarrow$ the current is lowest when $t = \frac{2}{3}$.

24. $20 = f(0) = \frac{a}{1+b}$ and $12 = f'(0) = -\frac{a}{(1+be^{-0.7t})^2}(be^{-0.7t})(-0.7)\Big|_{t=0} = \frac{a}{1+b} \cdot \frac{0.7b}{1+b} \Rightarrow$ $\frac{0.7b}{1+b} = 0.6$ and $\frac{a}{1+b} = 20 \Rightarrow a = 140$ and $b = 6$. Since $\lim\limits_{t\to\infty} f(t) = 140$, this means that the yeast population is near 140 in the long run.

30.
(a) $C'(100) = 25 - 0.18x + 0.0012x^2\big|_{x=100} = 19$, this gives the rate at which costs are increasing with respect to the production level when $x = 100$ and predicts the cost of the 101st item.
(b) The actual cost of 101st item is $C(101) - C(100) = 19.0304 \approx 19 = C'(100)$.

31.
(a) $A'(x) = \frac{xp'(x)-p(x)}{x^2}$. Since $A'(x) > 0$ implies that the average producitivity increases as new workers are added, the company want to hire more workers.
(b) $p'(x) > A(x) = \frac{p(x)}{x} \Rightarrow xp'(x) - p(x) > 0 \Rightarrow A'(x) > 0$.

35.
(a) Stable populations means that the growth rates of all species are 0, hence $\frac{dC}{dt} = 0$ and $\frac{dW}{dt} = 0$.
(b) $C = 0$.