

Section 3.3

Derivatives of trigonometric functions

1. $f(x) = 1 - 3 \sin x$

$$\Rightarrow f'(x) = (1)' - 3(\sin x)' = -3 \cos x$$

2. $f(x) = x \sin x$

$$\Rightarrow f'(x) = (x)' \sin x + x(\sin x)' = \sin x + x \cos x$$

9. $y = \frac{x}{2 - \tan x}$

$$\Rightarrow \frac{dy}{dx} = \frac{(2 - \tan x)(x)' - x(2 - \tan x)'}{(2 - \tan x)^2} = \frac{(2 - \tan x)(1) - x(-\sec^2 x)}{(2 - \tan x)^2} = \frac{2 - \tan x + x \sec^2 x}{(2 - \tan x)^2}$$

22. $y = e^x \cos x, \quad (0, 1)$

$$\frac{dy}{dx} = e^x \cos x + e^x(-\sin x) = e^x \cos x - e^x \sin x$$

$$\frac{dy}{dx}|_{x=0} = e^0 \cos 0 - e^0 \sin 0 = 1 - 0 = 1$$

The slope of the tangent line at $(0, 1)$ is 1. So, the equation of the tangent line at $(0, 1)$ is $y - 1 = 1(x - 0)$ or $y = x + 1$.

35. (a). $x(t) = 8 \sin t$

The equation of velocity $v(t) = x'(t) = 8 \cos t$.

The equation of acceleration $a(t) = x''(t) = -8 \sin t$.

(b). The mass at time $t = \frac{2\pi}{3}$ has position $x(\frac{2\pi}{3}) = 8 \sin(\frac{2\pi}{3}) = 8(\frac{\sqrt{3}}{2}) = 4\sqrt{3}$, velocity $v(\frac{2\pi}{3}) = 8 \cos(\frac{2\pi}{3}) = 8(\frac{-1}{2}) = -4$, and acceleration $a(\frac{2\pi}{3}) = -8 \sin(\frac{2\pi}{3}) = -8(\frac{\sqrt{3}}{2}) = -4\sqrt{3}$. Since $v(\frac{2\pi}{3}) < 0$, the particle is moving to the left.

37. From the figure 1: 3-3-12, we can see that $\sin \theta = x/6 \Leftrightarrow x = 6 \sin \theta$. We want to find the rate of change of x with respect to θ , that is $\frac{dx}{d\theta}$. Taking the derivative of $x = 6 \sin \theta$, we get $\frac{dx}{d\theta} = 6 \cos \theta$. So, when $\theta = \frac{\pi}{3}$, $\frac{dx}{d\theta} = 6 \cos(\frac{\pi}{3}) = 6(\frac{1}{2}) = 3 \text{ m/rd}$

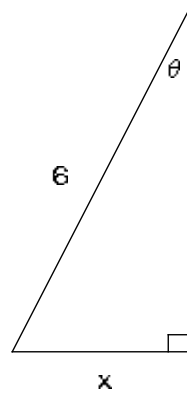


Figure 1: 3-3-12