1. Product Rule: $y = (x^2+1)(x^3+1) \Rightarrow$

$$y' = (x^2+1)(3x^2) + (x^3+1)(2x) = 3x^4 + 3x^2 + 2x^4 + 2x = 5x^4 + 3x^2 + 2x.$$

Multiplying first: $y = (x^2+1)(x^3+1) = x^5 + x^3 + x^2 + 1 \Rightarrow y' = 5x^4 + 3x^2 + 2x$ (equivalent).

3. By the Product Rule, $f(x) = (x^3+2x)e^x \Rightarrow$

$$\begin{aligned} f'(x) &= (x^3+2x)(e^x)' + e^x(x^3+2x)' = (x^3+2x)e^x + e^x(3x^2+2) \\ &= e^x[(x^3+2x)+(3x^2+2)] = e^x(x^3+3x^2+2x+2) \end{aligned}$$

4. By the Product Rule, $g(x) = \sqrt{x}\,e^x = x^{1/2}e^x \Rightarrow g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x+1)$.

5. By the Quotient Rule, $y = \dfrac{e^x}{x^2} \Rightarrow y' = \dfrac{x^2\frac{d}{dx}(e^x) - e^x\frac{d}{dx}(x^2)}{(x^2)^2} = \dfrac{x^2(e^x) - e^x(2x)}{x^4} = \dfrac{xe^x(x-2)}{x^4} = \dfrac{e^x(x-2)}{x^3}$.

11. $F(y) = \left(\dfrac{1}{y^2} - \dfrac{3}{y^4}\right)(y+5y^3) = (y^{-2} - 3y^{-4})(y+5y^3) \overset{\text{PR}}{\Rightarrow}$

$$\begin{aligned} F'(y) &= (y^{-2} - 3y^{-4})(1+15y^2) + (y+5y^3)(-2y^{-3} + 12y^{-5}) \\ &= (y^{-2} + 15 - 3y^{-4} - 45y^{-2}) + (-2y^{-2} + 12y^{-4} - 10 + 60y^{-2}) \\ &= 5 + 14y^{-2} + 9y^{-4} \text{ or } 5 + 14/y^2 + 9/y^4 \end{aligned}$$

14. $y = \dfrac{x+1}{x^3+x-2} \overset{\text{QR}}{\Rightarrow}$

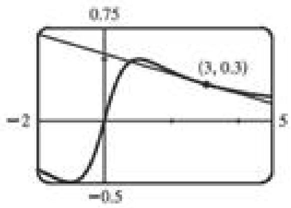
$$y' = \frac{(x^3+x-2)(1) - (x+1)(3x^2+1)}{(x^3+x-2)^2} = \frac{x^3+x-2-3x^3-3x^2-x-1}{(x^3+x-2)^2} = \frac{-2x^3-3x^2-3}{(x^3+x-2)^2}$$

or $-\dfrac{2x^3+3x^2+3}{(x-1)^2(x^2+x+2)^2}$

36. (a) $y = f(x) = \dfrac{x}{1+x^2} \Rightarrow$

$f'(x) = \dfrac{(1+x^2)1 - x(2x)}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$. So the slope of the tangent line at the point $(3, 0.3)$ is $f'(3) = \frac{-8}{100}$ and its equation is $y - 0.3 = -0.08(x-3)$ or $y = -0.08x + 0.54$.

(b)



43. We are given that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$.

(a) $(fg)'(5) = f(5)g'(5) + g(5)f'(5) = (1)(2) + (-3)(6) = 2 - 18 = -16$

(b) $\left(\dfrac{f}{g}\right)'(5) = \dfrac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \dfrac{(-3)(6) - (1)(2)}{(-3)^2} = -\dfrac{20}{9}$

(c) $\left(\dfrac{g}{f}\right)'(5) = \dfrac{f(5)g'(5) - g(5)f'(5)}{[f(5)]^2} = \dfrac{(1)(2) - (-3)(6)}{(1)^2} = 20$