

Solutions of 3.11

11. $\sinh x \cosh y + \cosh x + \sinh$

$$\begin{aligned} y &= \left[\frac{1}{2}(e^x - e^{-x})\right]\left[\frac{1}{2}(e^y + e^{-y})\right] + \left[\frac{1}{2}(e^x + e^{-x})\right]\left[\frac{1}{2}(e^y - e^{-y})\right] \\ &= \left[\frac{1}{4}(e^{x+y} + e^{x-y} - e^{-x+y} - e^{-x-y})\right] + \left[\frac{1}{4}(e^{x+y} - e^{x-y} + e^{-x+y} - e^{-x-y})\right] \\ &= \left[\frac{1}{2}(e^{x+y} - e^{-x-y})\right] = \sinh(x+y) \end{aligned}$$

38. $y' = \sinh x \cosh(\cosh x)$

54. $\lim_{x \rightarrow \infty} \frac{\sinh x}{e^x} = \lim_{x \rightarrow \infty} \frac{\frac{1-e^{-2x}}{2}}{1} = \frac{1}{2}$

55. $y' = \sinh x = 1 \Rightarrow e^{2x} - 2e^x - 1 = 0 \Rightarrow e^x = 1 + \sqrt{2}$

Ans: $(\ln(1 + \sqrt{2}), \sqrt{2})$

56. $\cosh x = \frac{1}{2} \left[\sec \theta + \tan \theta - \frac{1}{\sec \theta + \tan \theta} \right] = \frac{1}{2} \left[\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} \right] =$

$$\frac{1}{2} \left[\frac{\cos \theta + \sin \theta \cos \theta}{\cos^2 \theta} + \frac{\cos \theta - \sin \theta \cos \theta}{\cos^2 \theta} \right] = \sec \theta$$