Section3.10

11. Using the product rule and the chain rule

(a)

$$dy = (2x \sin 2x + x^2 \cos 2x \cdot 2)dx = [2x(\sin 2x + x \cos 2x)]dx$$

(b)

$$\frac{d}{dt} \ln \sqrt{1+t^2} = \frac{1}{\sqrt{1+t^2}} \frac{1}{2}(1+t^2)^{-\frac{1}{2}} 2t = \frac{t}{1+t^2} \Rightarrow dy = \frac{t}{1+t^2}dt$$

33. Similar as the example 4 in p.251.

(a)Define l = the edge of the cube.

$\because$ V = l^3

$\therefore \frac{d\mathrm{V}}{dl} = 3l^2 \Rightarrow d\mathrm{V} = 3l^2dl$

l =30cm, dl =0.1cm 代入

$\Rightarrow d\mathrm{V}$ =3(30)^{2}0.1 =270 cm^3

relative error = $\frac{d\mathrm{V}}{\mathrm{V}} = \frac{270}{30^3} = 0.01$ =1%

(b)Define A = the surface area.

A=6$l^2 \Rightarrow d$A= 12ldl

l=30cm, dl=0.1cm 代入 $\Rightarrow d$A=36cm^2

relative error = $\frac{d\mathrm{A}}{\mathrm{A}} = \frac{36}{6(30)^2} = 0.00\overline{6}$ =0.$\overline{6}$ %

40. 有關 Poiseuille's law 可以參考 http://www.answers.com/topic/poiseuille-s-law, 於 1836 實驗導出, 1940 發表. 不可壓縮之牛頓流體, 以層流狀態流經均勻圓管, 其流量F: 與半徑R 四次方, 壓力降ΔP 呈正比; 與粘度η, 管長L 呈反比。

solution:

$\because dF = 4kR^3dR$

$\therefore \frac{dF}{F} = \frac{4kR^3dR}{kR^4} = 4\frac{dR}{R}$

$\because$ the effect of 5% increase in the radius

$\therefore \frac{dF}{F}$ =4×5% = 20%

44. Appling the approximaiton formula in p.247

(a)$g(1.95) = g(2) + g'(2)(1.95-2) = -4.15$

$g(2.05) = g(2) + g'(2)(2.05-2) = -3.85$

(b) 因爲$y = g'(x)$的函數圖形是 convex(凹向上, 頂點在 x=0), 所以在 x=2附近,$g'(x)$(也就是$y = g(x)$圖型的斜率) 會隨著 x 增加而變大而且是正的, 所以$y = g(x)$的圖形在

$x=2$ 周圍是 convex(x 由2^-移動到2^+圖型斜率都是正的而且越來越大, 圖型類似p.247的FIGURE1), 故以 linear approximation 去做估計會造成低估 (too small)。