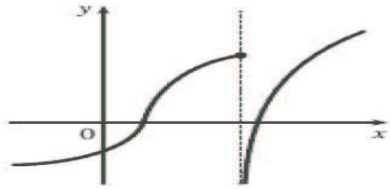
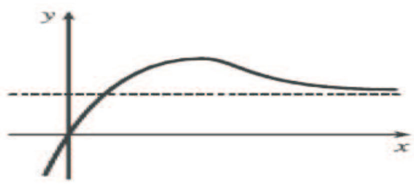
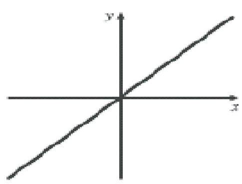
2. (a) The graph of a function can intersect a vertical asymptote in the sence that it can meet but not cross it.



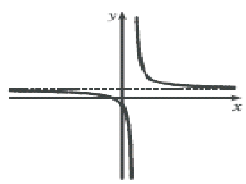
The graph of a function can intersect a horizontal asymptote. It can even intersect its horizontal asymptote an infinite number of times.



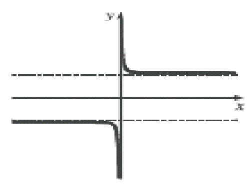
(b) The graph of a function can have 0, 1, or 2 hozizontal asymptotes. Representative examples are shown.



No horizontal asymptote



One horizontal asymptote



Two horizontal asymptotes

10. $\lim_{x\to 3} f(x) = -\infty$, $\lim_{x\to -\infty} f(x) = 2$, $f(0) = 0$, f is even. See figure 1.

19. Divide both the numerator and denominator by x^3 (the highest power of x that occurs in the denominator).

$$\lim_{x\to\infty} \frac{x^3+5x}{2x^3-x^2+4} = \lim_{x\to\infty} \frac{x^3+5x}{x^3}\,\frac{x^3}{2x^3-x^2+4} = \lim_{x\to\infty} \frac{1+\frac{5}{x^2}}{2-\frac{1}{x}+\frac{4}{x^3}} = \frac{\lim_{x\to\infty}(1+\frac{5}{x^2})}{\lim_{x\to\infty}(2-\frac{1}{x}+\frac{4}{x^3})}$$

$$= \frac{\lim_{x\to\infty} 1+5\lim_{x\to\infty}\frac{1}{x^2}}{\lim_{x\to\infty} 2-\lim_{x\to\infty}\frac{1}{x}+4\lim_{x\to\infty}\frac{1}{x^3}} = \frac{1+5(0)}{2-0+4(0)} = \frac{1}{2}.$$

23. $$\lim_{x\to\infty}\frac{\sqrt{9x^6-x}}{x^3+1} = \lim_{x\to\infty}\frac{\sqrt{9x^6-x}/x^3}{(x^3+1)/x^3}$$

$$= \frac{\lim_{x\to\infty}\sqrt{(9x^6-x)/x^6}}{\lim_{x\to\infty}(1+1/x^3)} \quad [\text{since } x^3 = \sqrt{x^6} \text{ for } x > 0]$$

$$= \frac{\lim_{x\to\infty}\sqrt{9-1/x^5}}{\lim_{x\to\infty} 1+\lim_{x\to\infty}(1/x^3)} = \frac{\sqrt{\lim_{x\to\infty} 9-\lim_{x\to\infty}(1/x^5)}}{1+0} = \sqrt{9-0} = 3.$$

41. $$\lim_{x\to\infty}\frac{2x^2+x-1}{x^2+x-2} = \lim_{x\to\infty}\frac{\frac{2x^+x-1}{x^2}}{\frac{x^2+x-2}{x^2}} = \lim_{x\to\infty}\frac{2+\frac{1}{x}-\frac{1}{x^2}}{1+\frac{1}{x}-\frac{2}{x^2}} = \frac{\lim_{x\to\infty}(2+\frac{1}{x}-\frac{1}{x^2})}{\lim_{x\to\infty}(1+\frac{1}{x}-\frac{2}{x^2})} = \frac{\lim_{x\to\infty} 2+\lim_{x\to\infty}\frac{1}{x}-\lim_{x\to\infty}\frac{1}{x^2}}{\lim_{x\to\infty} 1+\lim_{x\to\infty}\frac{1}{x}-\lim_{x\to\infty}\frac{2}{x^2}} = \frac{2+0-0}{1+0-2(0)} = 2,$$ so $y = 2$ is a horizontal asymptote.

$y = f(x) = \frac{2x^2+x-1}{x^2+x-2} = \frac{(2x-1)(x+1)}{(x+2)(x-1)}$, so $\lim_{x\to -2^-} f(x) = \infty$, $\lim_{x\to -2^+} f(x) = -\infty$, $\lim_{x\to 1^-} f(x) = -\infty$, and $\lim_{x\to 1^+} f(x) = \infty$. thus, $x = -2$ and $x = 1$ are vertical asymptotes. The graph confirms our work. See figure 2.

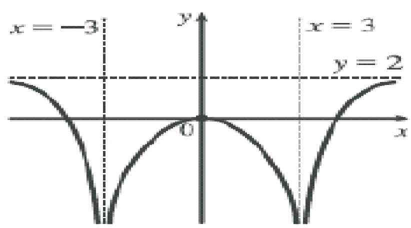


Figure 1: Exercise 10

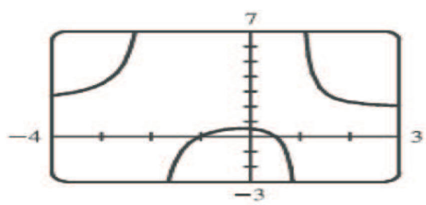


Figure 2: Exercise 41