

Section 2.5

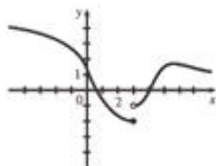
3. (a) The following are the numbers at which f is discontinuous and the type of discontinuity at that number: -4 (removable), -2 (jump), 2 (jump), 4 (infinite).

(b) f is continuous from the left at -2 since $\lim_{x \rightarrow -2^-} f(x) = f(-2)$. f is continuous from the right at 2 and 4 since

$$\lim_{x \rightarrow 2^+} f(x) = f(2) \text{ and } \lim_{x \rightarrow 4^+} f(x) = f(4). \text{ It is continuous from neither side at } -4 \text{ since } f(-4) \text{ is undefined.}$$

5. The graph of $y = f(x)$ must have a discontinuity at $x = 3$

and must show that $\lim_{x \rightarrow 3^-} f(x) = f(3)$.

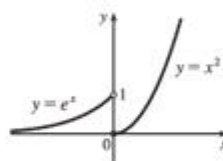


$$17. f(x) = \begin{cases} e^x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

The left-hand limit of f at $a = 0$ is $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} e^x = 1$. The

right-hand limit of f at $a = 0$ is $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2 = 0$. Since these

limits are not equal, $\lim_{x \rightarrow 0} f(x)$ does not exist and f is discontinuous at 0 .



$$39. f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ e^x & \text{if } 0 \leq x \leq 1 \\ 2-x & \text{if } x > 1 \end{cases}$$

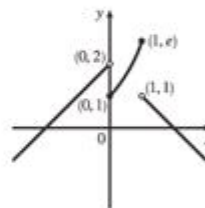
f is continuous on $(-\infty, 0)$ and $(1, \infty)$ since on each of these intervals

it is a polynomial; it is continuous on $(0, 1)$ since it is an exponential.

Now $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (x+2) = 2$ and $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} e^x = 1$, so f is discontinuous at 0 . Since $f(0) = 1$, f is

continuous from the right at 0 . Also $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} e^x = e$ and $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (2-x) = 1$, so f is discontinuous

at 1 . Since $f(1) = e$, f is continuous from the left at 1 .



$$41. f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^2 - cx & \text{if } x \geq 2 \end{cases}$$

f is continuous on $(-\infty, 2)$ and $(2, \infty)$. Now $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (cx^2 + 2x) = 4c + 4$ and

$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x^2 - cx) = 8 - 2c$. So f is continuous $\Leftrightarrow 4c + 4 = 8 - 2c \Leftrightarrow 6c = 4 \Leftrightarrow c = \frac{2}{3}$. Thus, for f

to be continuous on $(-\infty, \infty)$, $c = \frac{2}{3}$.

47. $f(x) = x^4 + x - 3$ is continuous on the interval $[1, 2]$, $f(1) = -1$, and $f(2) = 15$. Since $-1 < 0 < 15$, there is a number c in $(1, 2)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $x^4 + x - 3 = 0$ in the interval $(1, 2)$.