Section 2.4(Solutions)

3. Since $|\sqrt{x}-2| < 0.4 \Leftrightarrow 1.6 < \sqrt{x} < 2.4 \Leftrightarrow 2.56 < x < 5.76 \Leftrightarrow -1.44 < x-4 < 1.76$, so for any positive number $\delta \le 1.44$, we have $|x-4| < \delta \Rightarrow 2.56 \le 4-\delta < x < 4+\delta \le 5.44 \Rightarrow 2.56 < x < 5.76 \Rightarrow |\sqrt{x}-2| < 0.4$. Hence we can choose any δ such that $0 < \delta \le 1.44$.

4. Since $|x^2-1| < \frac{1}{2} \Leftrightarrow \frac{1}{2} < x^2 < \frac{3}{2} \Rightarrow \frac{\sqrt{2}}{2} < x < \frac{\sqrt{6}}{2} \Leftrightarrow \frac{\sqrt{2}}{2}-1 < x-1 < \frac{\sqrt{6}}{2}-1$, so for any positive number $\delta \le \frac{\sqrt{6}}{2}-1 = \min\{1-\frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{2}-1\}$, we have $|x-1| < \delta \Rightarrow \frac{\sqrt{2}}{2}-1 < x-1 < \frac{\sqrt{6}}{2}-1 \Rightarrow |x^2-1| < \frac{1}{2}$. Hence we can choose any δ such that $0 < \delta \le \frac{\sqrt{6}}{2}-1$.

19. Given any $\epsilon > 0$, let $\delta = 5\epsilon$. If $0 < |x-3| < \delta$, then $|\frac{x}{5}-\frac{3}{5}| = \frac{|x-3|}{5} < \frac{\delta}{5} = \epsilon$.

20. Given any $\epsilon > 0$, let $\delta = 4\epsilon$. If $0 < |x-6| < \delta$, then $|(\frac{x}{4}+3)-\frac{9}{2}| = \frac{|x-6|}{4} < \frac{\delta}{4} = \epsilon$.

21. Given any $\epsilon > 0$, let $\delta = \frac{5}{3}\epsilon$. If $0 < |x-(-5)| < \delta$, then $|(4-\frac{3x}{5})-7| = \frac{3|x+5|}{5} < \frac{3\delta}{5} = \epsilon$.

22. Given any $\epsilon > 0$, let $\delta = \epsilon$. If $0 < |x-3| < \delta$, then $|\frac{x^2+x-12}{x-3}-7| = |(x+4)-7| = |x-3| < \delta = \epsilon$.

23. Given any $\epsilon > 0$, let $\delta = \epsilon$. If $0 < |x-a| < \delta$, then $|x-a| < \epsilon$.

24. Given any $\epsilon > 0$, let $\delta = 1$. If $0 < |x-a| < \delta$, then $|c-c| = 0 < \epsilon$.

25. Given any $\epsilon > 0$, let $\delta = \sqrt{\epsilon}$. If $0 < |x-0| < \delta$, then $|x^2-0| = |x|^2 < \delta^2 = \epsilon$.

33. Given any $\epsilon > 0$, let $\delta = \min\{2, \frac{\epsilon}{8}\}$. If $0 < |x-3| < \delta$, then $|x+3| \le |x-3|+6 < \delta+6 \le 8$ by triangle inequality. Hence $|x^2-9| = |x+3||x-3| < 8\delta \le 8 \times \frac{\epsilon}{8} = \epsilon$.

41. $\frac{1}{(x+3)^4} > 10000 \Leftrightarrow |x+3| < 0.1$, hence the answer is 0.1.

42. Given any $M > 0$, let $\delta = \frac{1}{\sqrt[4]{M}} > 0$. If $0 < |x-(-3)| < \delta$, then $\frac{1}{(x+3)^4} > \frac{1}{\delta^4} = M$.