39. $|x-3| = \begin{cases} x-3 & \text{if } x-3 \geq 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases} = \begin{cases} x-3 & \text{if } x \geq 3 \\ 3-x & \text{if } x < 3 \end{cases}$

Thus, $\lim_{x\to 3^+}(2x+|x-3|) = \lim_{x\to 3^+}(2x+x-3) = \lim_{x\to 3^+}(3x-3) = 3(3)-3 = 6$ and

$\lim_{x\to 3^-}(2x+|x-3|) = \lim_{x\to 3^-}(2x+3-x) = \lim_{x\to 3^-}(x+3) = 3+3 = 6$. Since the left and right limits are equal,

$\lim_{x\to 3}(2x+|x-3|) = 6$.

40. $|x+6| = \begin{cases} x+6 & \text{if } x+6 \geq 0 \\ -(x+6) & \text{if } x+6 < 0 \end{cases} = \begin{cases} x+6 & \text{if } x \geq -6 \\ -(x+6) & \text{if } x < -6 \end{cases}$

We'll look at the one-sided limits.

$\lim_{x\to -6^+}\frac{2x+12}{|x+6|} = \lim_{x\to -6^+}\frac{2(x+6)}{x+6} = 2$ and $\lim_{x\to -6^-}\frac{2x+12}{|x+6|} = \lim_{x\to -6^-}\frac{2(x+6)}{-(x+6)} = -2$

The left and right limits are different, so $\lim_{x\to -6}\frac{2x+12}{|x+6|}$ does not exist.

41. $|2x^3-x^2| = |x^2(2x-1)| = |x^2|\cdot|2x-1| = x^2|2x-1|$

$|2x-1| = \begin{cases} 2x-1 & \text{if } 2x-1 \geq 0 \\ -(2x-1) & \text{if } 2x-1 < 0 \end{cases} = \begin{cases} 2x-1 & \text{if } x \geq 0.5 \\ -(2x-1) & \text{if } x < 0.5 \end{cases}$

So $|2x^3-x^2| = x^2[-(2x-1)]$ for $x < 0.5$.

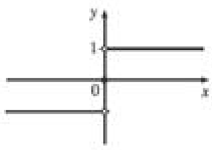
Thus, $\lim_{x\to 0.5^-}\frac{2x-1}{|2x^3-x^2|} = \lim_{x\to 0.5^-}\frac{2x-1}{x^2[-(2x-1)]} = \lim_{x\to 0.5^-}\frac{-1}{x^2} = \frac{-1}{(0.5)^2} = \frac{-1}{0.25} = -4$.

42. Since $|x| = -x$ for $x < 0$, we have $\lim_{x\to -2}\frac{2-|x|}{2+x} = \lim_{x\to -2}\frac{2-(-x)}{2+x} = \lim_{x\to -2}\frac{2+x}{2+x} = \lim_{x\to -2}1 = 1$.

43. Since $|x| = -x$ for $x < 0$, we have $\lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{|x|}\right) = \lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{-x}\right) = \lim_{x\to 0^-}\frac{2}{x}$, which does not exist since the denominator approaches 0 and the numerator does not.

44. Since $|x| = x$ for $x > 0$, we have $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{|x|}\right) = \lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{x}\right) = \lim_{x\to 0^+}0 = 0$.

45. (a)



(b) (i) Since $\operatorname{sgn} x = 1$ for $x > 0$, $\lim_{x\to 0^+}\operatorname{sgn} x = \lim_{x\to 0^+}1 = 1$.

(ii) Since $\operatorname{sgn} x = -1$ for $x < 0$, $\lim_{x\to 0^-}\operatorname{sgn} x = \lim_{x\to 0^-}-1 = -1$.

(iii) Since $\lim_{x\to 0^-}\operatorname{sgn} x \neq \lim_{x\to 0^+}\operatorname{sgn} x$, $\lim_{x\to 0}\operatorname{sgn} x$ does not exist.

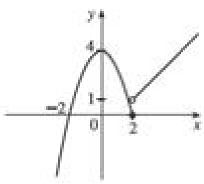
(iv) Since $|\operatorname{sgn} x| = 1$ for $x \neq 0$, $\lim_{x\to 0}|\operatorname{sgn} x| = \lim_{x\to 0}1 = 1$.

46. (a) $\lim_{x\to 2^-}f(x) = \lim_{x\to 2^-}(4-x^2) = \lim_{x\to 2^-}4 - \lim_{x\to 2^-}x^2 = 4-4 = 0$

$\lim_{x\to 2^+}f(x) = \lim_{x\to 2^+}(x-1) = \lim_{x\to 2^+}x - \lim_{x\to 2^+}1 = 2-1 = 1$

(b) No, $\lim_{x\to 2}f(x)$ does not exist since $\lim_{x\to 2^-}f(x) \neq \lim_{x\to 2^+}f(x)$.

(c)

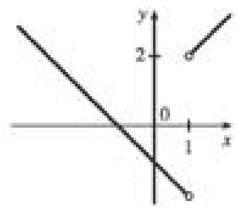


47. (a) (i) $\lim_{x\to 1^+} F(x) = \lim_{x\to 1^+} \frac{x^2-1}{|x-1|} = \lim_{x\to 1^+} \frac{x^2-1}{x-1} = \lim_{x\to 1^+} (x+1) = 2$

(ii) $\lim_{x\to 1^-} F(x) = \lim_{x\to 1^-} \frac{x^2-1}{|x-1|} = \lim_{x\to 1^-} \frac{x^2-1}{-(x-1)} = \lim_{x\to 1^-} -(x+1) = -2$

(b) No, $\lim_{x\to 1} F(x)$ does not exist since $\lim_{x\to 1^+} F(x) \neq \lim_{x\to 1^-} F(x)$.

(c)



49. (a) (i) $[\![x]\!] = -2$ for $-2 \le x < -1$, so $\lim_{x\to -2^+} [\![x]\!] = \lim_{x\to -2^+} (-2) = -2$

(ii) $[\![x]\!] = -3$ for $-3 \le x < -2$, so $\lim_{x\to -2^-} [\![x]\!] = \lim_{x\to -2^-} (-3) = -3$.

The right and left limits are different, so $\lim_{x\to -2} [\![x]\!]$ does not exist.

(iii) $[\![x]\!] = -3$ for $-3 \le x < -2$, so $\lim_{x\to -2.4} [\![x]\!] = \lim_{x\to -2.4} (-3) = -3$.

(b) (i) $[\![x]\!] = n-1$ for $n-1 \le x < n$, so $\lim_{x\to n^-} [\![x]\!] = \lim_{x\to n^-} (n-1) = n-1$.

(ii) $[\![x]\!] = n$ for $n \le x < n+1$, so $\lim_{x\to n^+} [\![x]\!] = \lim_{x\to n^+} n = n$.

(c) $\lim_{x\to a} [\![x]\!]$ exists $\Leftrightarrow$ a is not an integer.

57. Let $g(x)=0$, $h(x)=x^2$. Then $g(x)\le f(x) \le h(x)$ for all x. We know that $\lim_{x\to 0} g(x)=\lim_{x\to 0} h(x)=0$.

By Squeeze Theorem, $\lim_{x\to 0} f(x) = 0$.

60.

$$\lim_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1} = \lim_{x\to 2} \frac{(\sqrt{6-x}-2)(\sqrt{6-x}+2)(\sqrt{3-x}+1)}{(\sqrt{3-x}-1)(\sqrt{6-x}+2)(\sqrt{3-x}+1)} = \lim_{x\to 2} \frac{\sqrt{3-x}+1}{\sqrt{6-x}+2} = \frac{1}{2}$$

61. If such a exists, then $(x+2)|3x^2+ax+a+3$. That is, $3(-2)^2+a(-2)+a+3=0$. $a=15$.

$$\lim_{x\to -2} \frac{3x^2+15x+18}{x^2+x-2} = \lim_{x\to -2} \frac{(x+2)(3x+9)}{(x+2)(x-1)} = -1$$