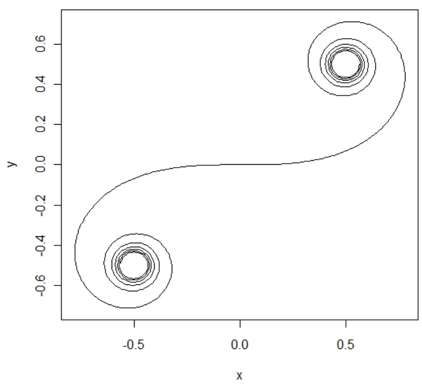
Section 10.2

56. (a) The curve is as follows:




When $t \to \infty$, the curve goes to the point $(0.5, 0.5)$ and when $t \to -\infty$, the curve goes to the point $(-0.5, -0.5)$.

(b) The arc length of the curve from $t = 0$ to $t = s$ is

$$\int_0^s \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^s \sqrt{(\cos(\pi t^2/2))^2 + (\sin(\pi t^2/2))^2} dt = \int_0^s dt = s$$

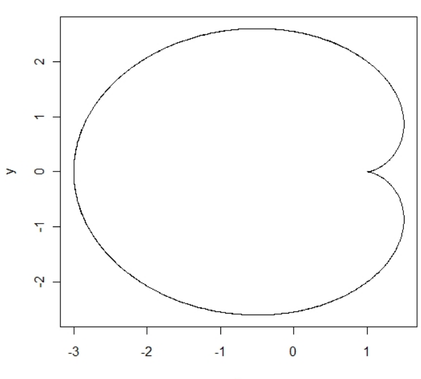
58. The area of the surface is

$$\int_0^{\frac{\pi}{3}} 2\pi \sin 3t \sqrt{\left(\frac{d\sin^2 t}{dt}\right)^2 + \left(\frac{d\sin 3t}{dt}\right)^2} dt = \int_0^{\frac{\pi}{3}} 2\pi \sin 3t \sqrt{4\sin^2 t \cos^2 t + 9\cos^2(3t)} dt \approx 7.477538$$

60. The area of the surface is

$$\int_0^1 3t^2 \sqrt{(3-3t^2)^2 + (6t)^2} dt = \int_0^1 3t^2(3t^2+3) dt = \int_0^1 9t^4 + 9t^2 dt = \frac{24}{5}$$

62. The curve is as follows:




The area of the surface is

$$\begin{aligned}
&\int_0^\pi 2\pi(2\sin\theta - \sin 2\theta)\sqrt{(-2\sin\theta + 2\sin 2\theta)^2 + (2\cos\theta - 2\cos 2\theta)^2} d\theta \\
=& \int_0^\pi 2\pi(2\sin\theta - \sin 2\theta)\sqrt{8 - 8\cos\theta} d\theta \\
=& \int_0^\pi 8\sqrt{2}\pi \sin\theta (1-\cos\theta)^{3/2} d\theta \\
=& 8\sqrt{2}\pi \int_0^\pi (1-\cos\theta)^{3/2} d(1-\cos\theta) = \frac{128\pi}{5}
\end{aligned}$$

64. The area of the surface is

$$\begin{aligned}
&\int_{\pi/4}^{\pi/2} 4\pi a \sin^2\theta \sqrt{(-2a\csc^2\theta)^2 + (4a\sin\theta\cos\theta)^2}d\theta \\
\approx\ & \frac{\pi}{8}(f(\pi/4) + 4f(3\pi/16) + 2f(3\pi/8) + 4f(5\pi/16) + f(\pi/2)) \\
\approx\ & 20.98558a^2
\end{aligned}$$

66. The area of the surface is

$$\begin{aligned}
&\int_0^1 2\pi(e^t - t)\sqrt{(e^t-1)^2 + (2e^{t/2})^2}dt \\
=\ & 2\pi\int_0^1 (e^t - t)(e^t + 1)dt \\
=\ & 2\pi\int_0^1 (e^t - t)(e^t - 1) + 2(e^t - t)dt \\
=\ & 2\pi\ \frac{1}{2}(e^t - t)^2 + 2\left(e^t - \frac{1}{2}t^2\right)\Big|_0^1 \\
=\ & \frac{e^2}{2} + e - 3
\end{aligned}$$

68.

$$S = \int_a^b 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}dx = \int_a^b 2\pi y\sqrt{1 + \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right)^2}\frac{dx}{dt}dt = \int_a^b 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}dt$$

70. (a)

$$\kappa = \frac{2}{(1+4x^2)^{3/2}}\Big|_{x=1} = \frac{2\sqrt{5}}{25}$$

(b)

$$\frac{d\kappa}{dx} = -3(1+4x^2)^{-\frac{5}{2}} \cdot 8x = 0 \Rightarrow x = 0$$

and $\kappa$ increases on $(-\infty, 0)$ since $\frac{d\kappa}{dx} > 0$ for $x < 0$ and decreases on $(0,\infty)$ since $\frac{d\kappa}{dx} < 0$ for $x > 0$. Thus when $x = 0$, $\kappa$ reaches its maximum.

72. (a) Note that the parametric expression for a straight line must be $\begin{cases} x = a + bt \\ y = c + dt \end{cases}$.

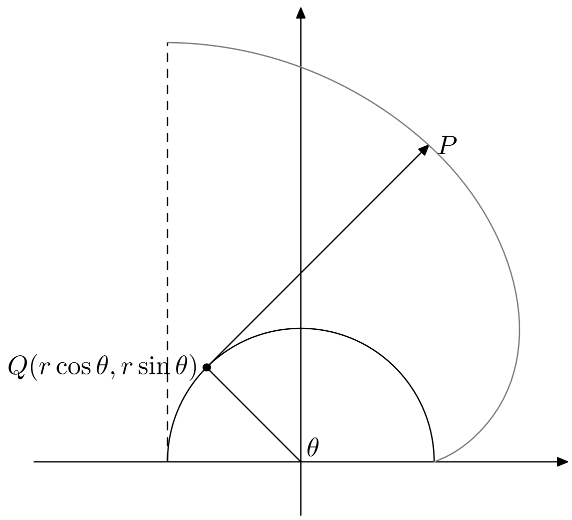
Hence $\ddot{x} = \ddot{y} = 0$ and $|\dot{x}\ddot{y} - \dot{y}\ddot{x}| = 0$.
Thus the curvature must be 0.

(b) Note that the parametric expression for a straight line must be $\begin{cases} x = r\cos t + a \\ y = r\sin t + b \end{cases}$ where $r$ is the radius.

Hence $\dot{x} = -r\sin t$, $\dot{y} = r\cos t$, $\ddot{x} = -r\cos t$, $\ddot{y} = -r\sin t$ and

$$\kappa = \frac{|r^2\sin^2 t + r^2\cos^2 t|}{[r^2\sin^2 t + r^2\cos^2 t]^{3/2}} = \frac{1}{r}$$

74. From the figure below, the slope of $\overline{QP}$ is $\tan\left(\theta - \frac{\pi}{2}\right) = -\cot\theta$ and the lenght is $r\theta$.




Thus the coordinate of $P$ is $(r\cos\theta, r\sin\theta) + r\theta(\sin\theta, -\cos\theta)$.

And the area below the curve is

$$\left|\int y dx\right| = \left|\int_0^\pi (r\sin\theta - r\theta\cos\theta) r\theta\cos\theta d\theta\right| = \frac{(3\pi+\pi^3)r^2}{6}$$

and the area we want is

$$2\left(\frac{(3\pi+\pi^3)r^2}{6} - \frac{\pi r^2}{2} + \frac{\pi(\pi r)^2}{4}\right) = \frac{5\pi^3 r^2}{6}$$