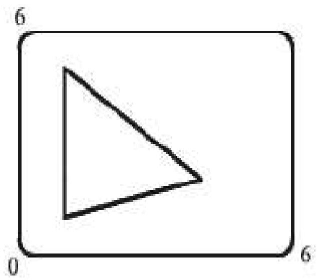
32.



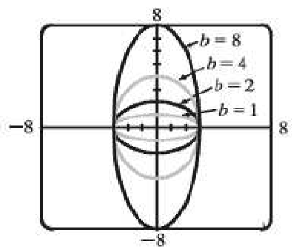
For the side of triangle from A to B, use $(x_1, y_1) = (1, 1)$ and $(x_2, y_2) = (4, 2)$. Hence, the equation are

$$x = x_1 + (x_2 - x_1)t = 1 + (4 - 1)t = 1 + 3t,$$

$$y = y_1 + (y_2 - y_1)t = 1 + (2 - 1)t = 1 + t.$$

Graph $x = 1+3t$ and $y = 1+t$ with $0 \leq t \leq 1$ gives us the side of the triangle from A to B. Similarly, for the side BC we use $x = 4 - 3t$ and $y = 2 + 3t$, and for the side AC we use $x = 1$ and $y = 1 + 4t$.

34.



(a) Let $\frac{x^2}{a^2} = sin^2 t$ and $\frac{y^2}{b^2} = cos^2 t$ to obtain $x = asint$ and $y = bcost$ with $0 \leq t \leq 2\pi$ as possible parametric equations for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
(b) The equation are $x = 3sint$ and $y = bcost$ for $b \in \{1, 2, 4, 8\}$.
(c) As b increase, the ellipse stretches vertically.

36. If you are using a calculator or computer that can overlay graphs(using multiple $t-$intervals), the following are appropriate.
$Leftside$: $x = 1$ and y goes from 1.5 to 4, so use

$$x = 1, \qquad y = t, \qquad 1.5 \leq t \leq 4$$

$Rightside$: $x = 10$ and y goes from 1.5 to 4, so use

$$x = 10, \qquad y = t, \qquad 1.5 \leq t \leq 4$$

$Bottom$: x goes from 1 to 10 and $y = 1.5$, so use

$$x = t, \qquad y = 1.5, \qquad 1 \leq t \leq 10$$

$Handle$: It starts at (10,4) and ends at (13,7), so use

$$x = 10 + t, \qquad y = 4 + t, \qquad 0 \leq t \leq 3$$

$Leftwheel$:It's centered at (3,1),has radius of 1,and appears to go about $30°$ above the horizontal, so use

$$x = 3 + cost, \qquad y = 1 + sint, \qquad \frac{5\pi}{6} \leq t \leq \frac{13\pi}{6}$$

$Rightwheel$:Similar to the left wheel with center (8,1), so use

$$x = 8 + cost, \qquad y = 1 + sint, \qquad \frac{5\pi}{6} \leq t \leq \frac{13\pi}{6}$$

If you are using a calculator or computer that cannot overlay graphs (using one $t-$interval), the following is appropriate. We'll start by picking the $t-$interval $[0, 2.5]$ since it easily matches the $t-$values for the two sides. We now need to find parameter equations for all graphs with $0 \leq t \leq 2.5$.

$Leftside$: $x = 1$ and y goes from 1.5 to 4, so use

$$x = 1, \qquad y = 1.5 + t, \qquad 0 \leq t \leq 2.5$$

$Rightside$: $x = 10$ and y goes from 1.5 to 4, so use

$$x = 10, \qquad y = 1.5 + t, \qquad 0 \leq t \leq 2.5$$

Bottom : x goes from 1 to 10 and $y = 1.5$, so use

$$x = 1 + 3.6t, \qquad y = 1.5, \qquad 0 \le t \le 2.5$$

To get the $x-$assignment, think of creating a linear function such that $t = 0, x = 1$ and when $t = 2.5,\ x = 10$. We can use the point-slope form of a line with $(t_1, x_1) = (0, 1)$ and $(t_2, x_2) = (2.5, 10.)$

$$x - 1 = \frac{10 - 1}{2.5 - 0}(t - 0) \Rightarrow x = 1 + 3.6t.$$

Handle : It starts at (10,4) and ends at (13,7), so use

$$x = 10 + 1.2t, \qquad y = 4 + 1.2t, \qquad 0 \le t \le 2.5$$

$(x_1, t_1) = (0, 10)$ and $(t_2, x_2) = (2.5, 13)$ gives us $x - 10 = \frac{13-10}{2.5-0}(t - 0) \Rightarrow x = 10 + 1.2t$
$(t_1, y_1) = (0, 4)$ and $(t_2, y_2) = (2.5, 7)$ gives us $y - 4 = \frac{7-4}{2.5-0}(t - 0) \Rightarrow y = 4 + 1.2t$

Leftwheel :It's centered at (3,1),has radius of 1,and appears to go about 30° above the horizontal, so use

$$x = 3 + cos(\frac{8\pi}{15}t + \frac{5\pi}{6}), \qquad y = 1 + sin(\frac{8\pi}{15}t + \frac{5\pi}{6}), \qquad 0 \le t \le 2.5$$

$(t_1, \theta_1) = (0, \frac{5\pi}{6})$ and $(t_2, \theta_2) = (\frac{5}{2}, \frac{13\pi}{6})$ gives us $\theta - \frac{5\pi}{6} = \frac{\frac{13\pi}{6} - \frac{5\pi}{6}}{\frac{5}{2} - 0}(t - 0) \Rightarrow \theta = \frac{8\pi}{15}t + \frac{5\pi}{6}$.

Rightwheel :Similar to the left wheel with center (8,1), so use

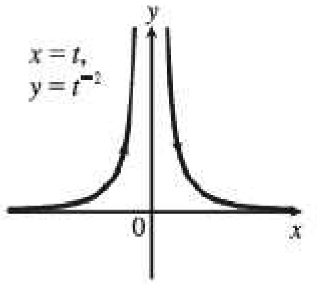
$$x = 8 + cos(\frac{8\pi}{15}t + \frac{5\pi}{6}), \qquad y = 1 + sin(\frac{8\pi}{15}t + \frac{5\pi}{6}), \qquad 0 \le t \le 2.5$$

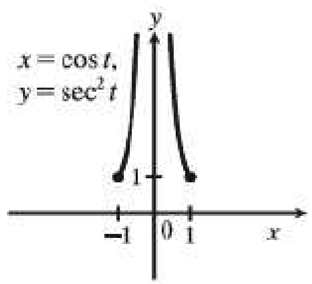
38.

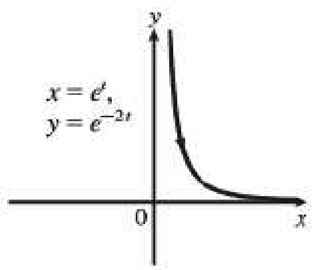
(a)$x = t$, so $y = t^{-2} = x^{-2}$. We get the entire curve $y = \frac{1}{x^2}$ travesed in a left-to-right direction.

(b) $x = cost$, $y = sec^2 t = \frac{1}{cos^2 t} = \frac{1}{x^2}$. Since $sect \ge 1$, we only get the parts of the curve $y = \frac{1}{x^2}$ with $y \ge 1$. We get the first quadrant portion of the curve when $x > 0$, that is $cost > 0$, and we get the second quadrant portion of the curve when $x < 0$, that is , $cost < 0$.

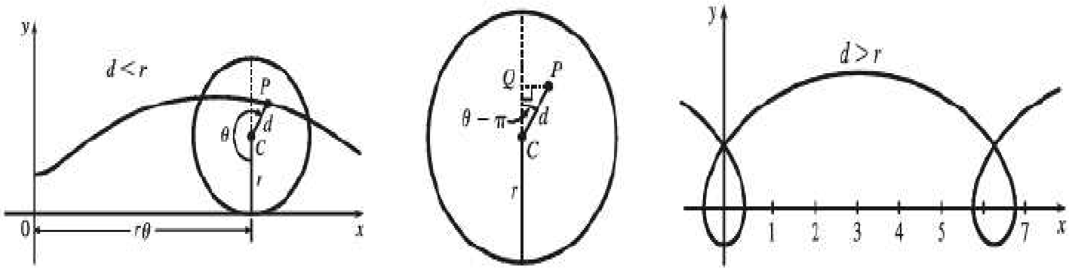
(c)$x = e^t,\ y = e^{-2t} = x^{-2}$. Since e^t and e^{-2t} are both positive, we only get the first quadrant portion of the curve $y = \frac{1}{x^2}$.
So we list the graphs as follows:







40.
The first two diagram depict the case $\pi < \theta < \frac{3\pi}{2}$, $d < r$. As in Example6, C has coordinates $(r\theta, r)$. Now Q(in the second diagram) has coordinates



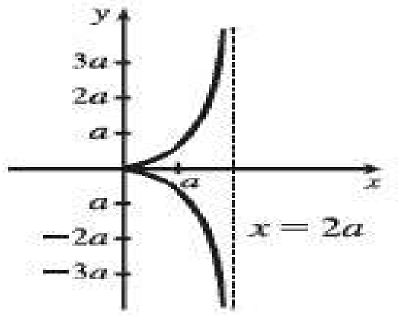
$(r\theta, r + dcos(\theta - \pi)) = (r\theta, r - dcos\theta)$, so a typical point P of the trochoid has coordinates $(r\theta + dsin(\theta - \pi), r - dcos\theta)$. That is, P has coordinates (x, y), where $x = r\theta - dsin\theta$ and $y = r - dcos\theta$. When $d = r$, these equations agree with those of the cycloid.

42.
A has coordinates $(acos\theta, asin\theta)$. Since OA is perpendicular to AB, $\triangle OAB$ is right triangle and B has coordinates $(asec\theta, 0)$. It follows that P has coordinates $(asec\theta, bsin\theta)$. Thus, the parametric equations are $x = asec\theta, y = bsin\theta$.

44.
(a)Let θ be the angle of inclination of segment OP. Then $|OB| = \frac{2a}{cos\theta}$. Let $C = (2a, 0)$. Then by use of triangle OAC we see that $|OA| = 2acos\theta$. Now

$$|OP| = |AB| = |OB| - |OA| = 2a(\frac{1}{cos\theta} - cos\theta) = 2a\frac{1 - cos^2\theta}{cos\theta} = 2a\frac{sin^2\theta}{cos\theta} = 2asin\theta cos\theta.$$

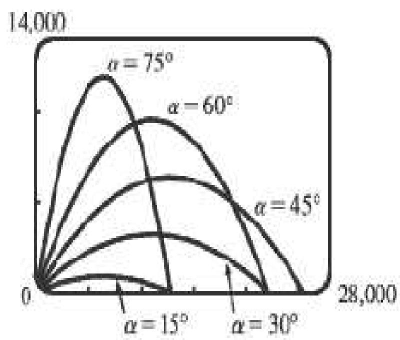
(b)



46.
(a) If $\alpha = 30^\circ$ and $v_0 = 500m/s$, then the equations become $x = (500cos30^\circ)t = 250\sqrt{3}t$ and $y = (500sin30^\circ)t - \frac{1}{2}(9.8)t^2 = 250t - 4.9t^2$. $y = 0$ when $t = 0$(when gun is fired) and again when $t = \frac{250}{4.9} \approx 51s$. Then $x = (250\sqrt{3})(\frac{250}{4.9}) \approx 22092m$, so the bullet hits the ground about 22 km from the gun. The formula for y is quadratic in t. To find the maximum $y-$value, we will complete the square:

$$y = -4.9(t^2 - \frac{250}{4.9}t) = -4.9(t - \frac{125}{4.9})^2 + \frac{125^2}{4.9} \leq \frac{125^2}{4.9}$$

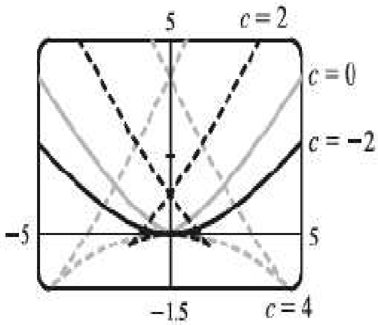
with equality when $t = \frac{125}{4.9}s$, so the maximum height attained is $\frac{125^2}{4.9} \approx 3189m$.
(b)



As $\alpha(0^\circ < \alpha < 90^\circ)$ increases up to 45°, the projectile attains a greater

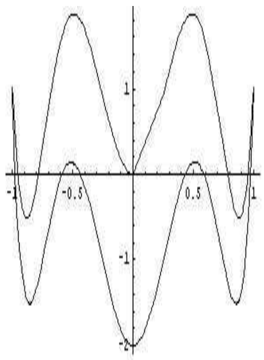
height and a greater range. As α increases pass $45°$, the projectile attains a greater height, but its range decreases.
(c) $x = (v_0 cos\alpha)t \Rightarrow t = \frac{x}{v_0 cos\alpha}$. So $y = (v_0 sin\alpha)t - \frac{1}{2}gt^2 \Rightarrow y = (v_0 sin\alpha)\frac{x}{v_0 cos\alpha} - \frac{g}{2}(\frac{x}{v_0 cos\alpha})^2 = (tan\alpha)x - (\frac{g}{2v_0^2 cos^2\alpha})x^2$ which is the equation of a parabola (quadratic in x).

48.

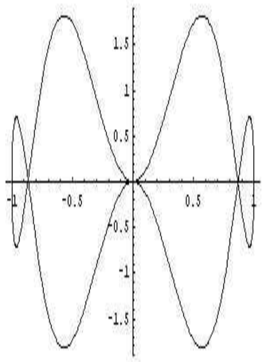


$x = 2ct - 4t^3$, $y = -ct^2 + 3t^4$. We use a graphing device to produce the graphs for various values of c with $-\pi \leq t \leq \pi$. Note that all the members of the family are symmetric about $y-$axis. When $c < 0$, the graph resembles that of a polynomial of even degree, but when $c = 0$ there is a corner at the origin, and when $c > 0$, the graph crosses itself at the origin, and has two cusps below the $x-$axis. The size of the "swallowtail" increases as c increases.

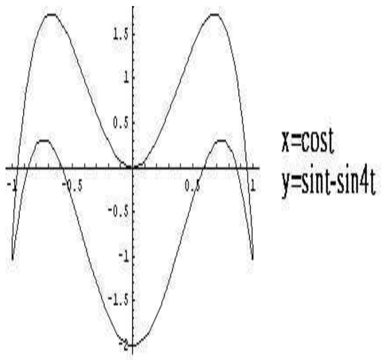
50. We plot the graphs of the cases $c = 2, 3, 4, 5, 6, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$. When $c \in \mathbb{Z}$, we can divide those into two parts, that are, even and odd cases. When $c \in \mathbb{Q}$, their graphs are similar. List those as following:

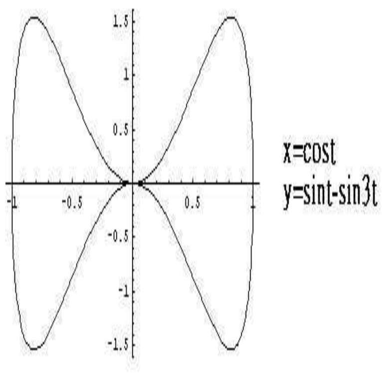


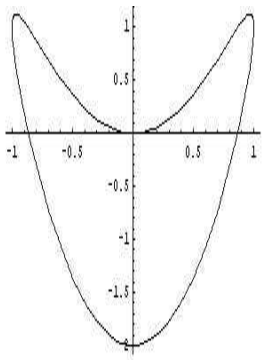
x=cost
y=sint-sin6t



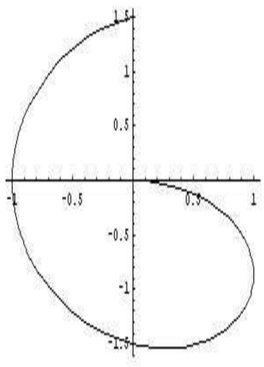
x=cost
y=sint-sin5t


x=cost
y=sint-sin4t

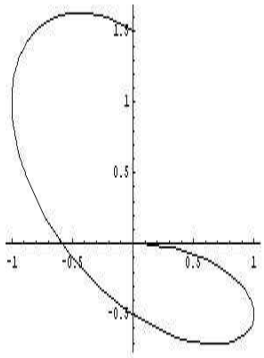

x=cost
y=sint-sin3t



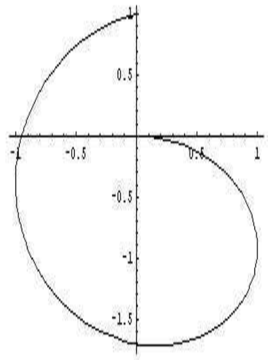
x=cost
y=sint-sin2t



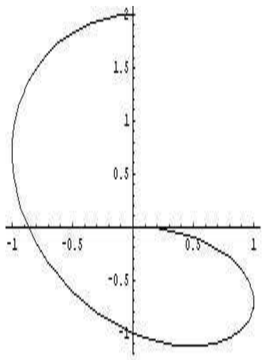
x=cost
y=sint-sint/3



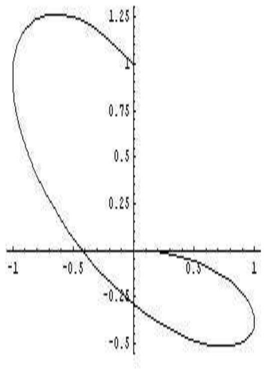
x=cost
y=sint-sin2t/3



x=cost
y=sint/4



x=cost
y=sint-sint/2



x=cost
y=sint-sin3t/4