92學年上學期微積分甲統一教學 I 組期中考題目卷 (單面, 共9大題)

1. (a) $\lim_{x\to\frac{\pi}{4}} \tan 2x \cdot \tan(\frac{\pi}{4} - x)$. (10points)

(b) Suppose $\lim_{x\to\infty} f'(x) = A$, $a \neq 0$, find $\lim_{x\to\infty}\{f(x+a) - f(x)\}$. (10points)

2. Suppose $f(x) = \begin{cases} x^2 & x \le 1 \\ ax+b & x > 1 \end{cases}$. Find a and b such that f is continuous and differentiable at $x = 1$. (10points)

3. Find the equation of the tangent line of the graph: $y^2 = x^3 + 3x^2$ at $(1, -2)$. (5points)

4. Graph $y = \frac{x+1}{\sqrt{|x-1|}}$. Be sure to compute intervals of monotonicity, the intervals of concavity, the position of all local extrema, and inflection points, and all asymptotic lines. (15points)

5. Find $\frac{d^n}{dx^n}(\frac{1+x}{\sqrt{1-x}})$.(10points)

6. Given a sphere with radius r, find the height h of a pyramid of minimum volume whose base is a square and whose faces are all tengent to the sphere. (10points)

7. (a)$\int \frac{x+1}{\sqrt{2x+1}}dx$. (5points)

(b)$\int_0^\pi \sqrt{\sin^3 x - \sin^5 x}dx$. (5points)

8. Prove or disprove

(a)If $x^3 + x = f(x)$ has at least two solutions, then there exists t such that $f'(t) \ge 1$. (5points)

(b)We can find a non-constant differentiable function f defined for all x such that $f'(x) = 0$ for all $\frac{1}{n}$, n=1,2,3,... (5points)

9. Let $f(x)$ be a continuous function.

Define

$$F(x) = \int_0^x (\int_0^{u^2} f(t)dt)du \text{ for } x \ge 0$$

$$G(x) = \int_0^{x^2} f(u)(x - \sqrt{u})du \text{ for } x \ge 0$$

Compute $F'(x)$ and $G'(x)$, and prove that $F(x) = G(x)$ for $x \ge 0$. (10points)