92學年上學期微甲一組期末考

1. The curve $y^2 = x\left(\frac{1}{3} - x\right)^2$ encloses a bounded plane region. Find the area of this plane region (5%) and the arc length of the boundary curve (5%). Now we revolve this plane region around y-axis to obtain a solid of revolution. Find the volume of this solid (5%).

2. Find the limit $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\tan^2 x}\right)$. (10%)

3. Find the derivative $\frac{d}{dx}\left[x \sec^{-1}(2x) - \frac{1}{4}\ln\left(1 + 4x^2\right)\right]$. (10%)

4. Find the antiderivative $\int \frac{e^x + 1}{e^{2x} - e^x + 2} dx$. (10%)

5. Evaluate the integral $\int_0^{\frac{\pi}{2}} \sin^n \theta \, d\theta$, $n \in \mathbb{N}$. (10%)

6. Find the improper integral $\int_0^1 \frac{1}{\sqrt{x(1-x)}} \, dx$. (10%)

7. An observer at height H above the north pole of a sphere of radius r. Find the surface area that he can see. (10%)

8. Test the series for convergence or divergence.

   (a) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{n+1}\right)^{n^2}$. (5%)

   (b) $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n} - 1}$. (5%)

   (c) $\sum_{n=1}^{\infty} \frac{n}{n^2 + 3}$. (5%)

9. Find the Taylor series expansion of the function e^{x^2} about $x = 0$ and prove that it converges to e^{x^2}. (10%)