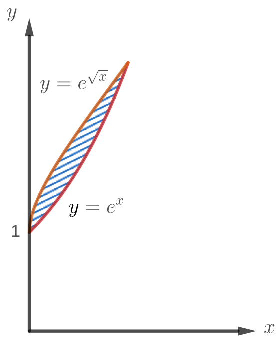
1. (15 pts) Let $f(x,y,z) = \sin(xy+z)$, and P be the point $(0,-2,\frac{\pi}{3})$.
   (a) (6 pts) Compute $\nabla f(x,y,z)$.
   (b) (2 pts) At P, find the direction along which f obtains maximum directional derivative.
   (c) (4 pts) Calculate the directional derivative $\frac{\partial f}{\partial \mathbf{u}}(P)$, where $\mathbf{u}$ is a unit vector making an angle $\frac{\pi}{6}$ with the gradient $\nabla f(P)$.
   (d) (3 pts) The level surface $f(x,y,z) = \frac{\sqrt{3}}{2}$ defines z implicitly as a function of x and y near P. Compute $\frac{\partial z}{\partial x}$ at P.

2. (12 pts) Assume that $f(x,y,z)$ and $g(x,y,z)$ have continuous partial derivatives and $(1,2,-1)$ lies on the level surface $f(x,y,z) = 3$. Suppose the tangent plane of $f(x,y,z) = 3$ at $(1,2,-1)$ is $2x - y + 3z + 3 = 0$ and $f_y(1,2,-1) = 2$.
   (a) (4 pts) Find $\nabla f(1,2,-1)$.
   (b) (4 pts) Estimate $f(1.1, 2.01, -0.98)$ by the linear approximation of f at $(1,2,-1)$.
   (c) (4 pts) Suppose that when restricted on the surface $f(x,y,z) = 3$, $g(x,y,z)$ obtains maximum value at $(1,2,-1)$ and $g_x(1,2,-1) = -2$. Find $\nabla g(1,2,-1)$ and the maximum directional derivative of g at the point $(1,2,-1)$.

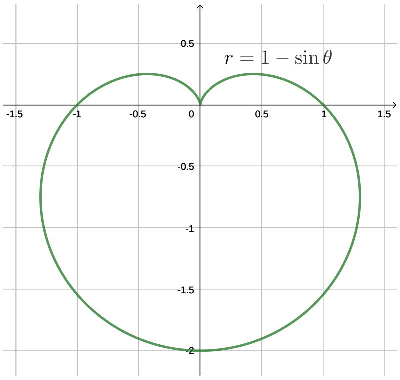
3. (25 pts) $f(x,y) = x^2 + xy + y^2 + 3x$.
   (a) (7 pts) Find critical point(s) of $f(x,y)$ and determine whether it is a saddle point or $f(x,y)$ obtains local maximum or local minimum at it.
   (b) (15 pts) Find the maximum and minimum value of $f(x,y)$ on the curve $x^2 + y^2 = 9$ by the method of Lagrange multipliers.
   (c) (3 pts) Find the maximum value of $f(x,y)$ on the region $x^2 + y^2 \le 9$.

4. (18 pts) (a) (8 pts) Reverse the order of integration and evaluate it. $\int_0^4 \int_{\frac{\sqrt{y}}{2}}^1 \sqrt{x^3+3}\,dx\,dy$.
   (b) (10 pts) Compute $\iint\limits_{\Omega} (\ln y)^{-1} dA$, where Ω is bounded by $y = e^x$ and $y = e^{\sqrt{x}}$.



5. (18 pts) (a) (8 pts) Evaluate $\iint\limits_{D} e^{-x^2-y^2} dA$, where D is the upper disc, $x^2 + y^2 \le 25$ and $y \ge 0$.
   (b) (10 pts) Calculate the area of the region inside the cardioid

$$r = 1 - \sin\theta.$$



6. (12 pts) Evaluate $\displaystyle\iint_D e^{xy}\,dxdy$, where D is bounded by curves $xy = 10$, $xy = 20$, $x^2y = 20$ and $x^2y = 40$.