1. (22%) Compute the following limits. (You can not use l'Hospital's Rule.)

   (a) (4%) $\lim_{x\to-\infty} \dfrac{e^{-x}+2}{3e^{x}-e^{-x}}$.

   (b) (6%) $\lim_{x\to\infty} \sqrt{x^2-x}-x+\tan^{-1}x$.

   (c) (6%) $\lim_{x\to0} \dfrac{\cos x-1}{x^2}$.

   (d) (6%) $\lim_{x\to1} \dfrac{\ln x}{x^3-1}$.

2. (20%) (a) (6%) Let $f(x)=\dfrac{x}{3x^2+4}$. Calculate $f'(x)$.

   (b) (7%) Let $f(x)=\tan^{-1}(e^x)+\sin^{-1}(\cos(2x))$. Calculate $f'(x)$.

   (c) (7%) Let $f(x)=x^{\sin x}$. Calculate $f'(\pi)$.

3. (10%) Consider the plane curve given by the equation $x^3+xy+\frac{y^3}{8}=4$.

   (a) (6%) Compute $\frac{dy}{dx}$ at the point $(1,2)$.

   (b) (4%) The curve near the point $(1,2)$ can be described as $y=f(x)$. Use the linear approximation to estimate $f(1.01)$.

4. (16%) Let $f(x)=\tan x$, $x\in[0,\frac{\pi}{2})$.

   (a) (6%) Show that $\tan x>x$ for $x\in(0,\frac{\pi}{2})$.

   (b) (4%) Using the result from (a), explain that $f'(x)=1+f^2(x)>1+x^2$, for $x\in(0,\frac{\pi}{2})$.

   (c) (6%) Show that $\tan x>x+\frac{1}{3}x^3$ for $x\in(0,\frac{\pi}{2})$.

5. (20%) Sketch the graph of the function $f(x)=\ln|x+1|+5x+x^2$.

   (a) (2%) Write down the domain of $f(x)$. Find $\lim_{x\to-1^-} f(x)$ and $\lim_{x\to-1^+} f(x)$.

   (b) (7%) Compute $f'(x)$. Find the interval(s) of increase and interval(s) of decrease of $f(x)$. Find local extreme values of $f(x)$.

   (c) (7%) Compute $f''(x)$. Determine the concavity of $y=f(x)$ and find the inflection point(s).

   (d) (4%) Sketch the graph of $f(x)$.

6. (12%) A lamp is hung over the center of a circular table with radius 2m. Find the height h of the lamp over the table such that the illumination I at the perimeter of the table is maximum. We know that $I=\frac{k\sin\alpha}{s^2}$, where $k>0$ is a constant, s is the slant length, and α is the angle at which the light strikes the perimeter. (You need to explain that the extreme value you find is the absolute maximum.)