

1. (14%) Look for critical points of  $f(x, y) = \frac{x}{x^2 + y^2 + 1}$  and determine their extremum properties.
2. (15%)  $x^2 + y^2 = 1$ . Use Lagrange multiplier method to find the maximum and minimum values of  $x^2y - y^2$ .
3. (14%) Let  $F(u, v) = \left(u, \frac{v}{u+1}\right)$  and  $\Sigma$  be the parallelogram with vertices

$$(0, 0), \quad (2, 0), \quad (1, 1), \quad (3, 1)$$

in the  $uv$ -plane. Suppose that  $\Omega = \{F(u, v) : (u, v) \in \Sigma\}$  is the image of  $\Sigma$  via the map  $F$ . Evaluate the area of  $\Omega$  (recall that  $\int \ln x \, dx = x \ln x - x + c$ ).

4. (15%)  $\Omega$  is the region bounded by the loop of  $r^2 = \cos 2\theta$  in the first and fourth quadrants. Evaluate  $\iint_{\Omega} x^2 + y^2 \, dA$ .

