1. (20%) Find the following limits.

   (a) (5%) $\lim_{x\to 1} \dfrac{\sqrt{2x^2+3x-4}-x}{x-1}$. (b) (5%) $\lim_{x\to 0^-} x\cdot\sqrt{1+\dfrac{9}{x^2}}$.

   (c) (5%) $\lim_{x\to 0} \dfrac{\cos 3x-1}{x^2}$. (d) (5%) $\lim_{x\to\infty}\left(1+\dfrac{e}{x}\right)^{2x}$. (Hint: $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^{x}=e$.)

2. (15%) Find the derivative of the function.

   (a) (5%) $f(x)=\dfrac{\sin 2x}{1-\cos 2x}$. (b) (5%) $f(x)=\tan^{-1}\left(\sqrt{e^x-1}\right)$.

   (c) (5%) $f(x)=(\sec x)^x$, $-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}$.

3. (14%) $f(x)=x^5+2x-3$.

   (a) (4%) Explain that $f(x)$ is 1-1.

   (b) (5%) We know that $f(x)$ has inverse function from (a). Find $f^{-1}(-3)$ and $(f^{-1})'(-3)$.

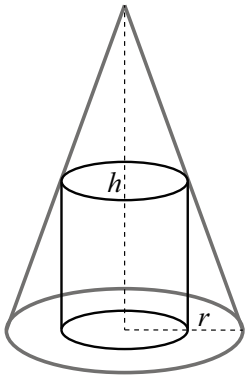
   (c) (5%) Write down the linear approximation of $f^{-1}(x)$ at $x=-3$. Estimate $f^{-1}(-3.01)$ by the linear approximation.

4. (9%) Find the equation of the tangent line to the curve $\ln(x^2-3y)=x-y-1$ at the point $(2,1)$.

5. (6%) By the Mean Value Theorem, explain that

$$\frac{a}{\sqrt{1-a^2}}<\sin^{-1}(2a)-\sin^{-1}(a)<\frac{a}{\sqrt{1-4a^2}},\ \text{where } 0<a<\frac{1}{2}.$$

6. (12%) For a right cone with base radius r and height h, $r,h>0$, find the inscribed right cylinder (as figure) of maximal volume.



7. (24%) Follow the steps to sketch the graph of the function $f(x)=\dfrac{1}{x}-\dfrac{1}{3x^3}$.

   (a) (2%) Discuss the symmetry of $y=f(x)$.

   (b) (6%) Compute $f'(x)$. Find interval(s) of increase and interval(s) of decrease of $f(x)$.

   (c) (2%) Classify (local) extreme values.

   (d) (6%) Compute $f''(x)$. Discuss concavities of $y=f(x)$.

   (e) (2%) Find inflection point(s) of $y=f(x)$.

   (f) (2%) Find asymptotes of $y=f(x)$.

   (g) (4%) Sketch the graph of $f(x)$.