1. (10%) Let $F(x) = \int_0^{x^2} \cos(t^2 + t)\,\mathrm{d}t$.

   (a) (6%) Compute $F'(x)$.

   (b) (4%) Compute the limit $\lim_{x\to 0} \dfrac{F(x)}{x^2}$.

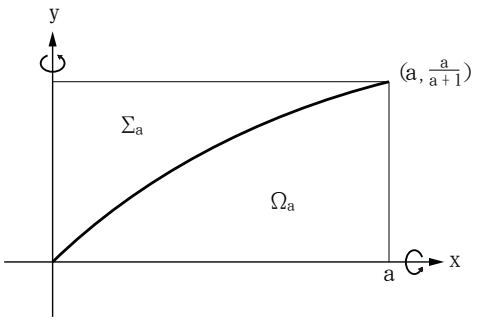
2. (14%) Compute the integrals.

   (a) (7%) $\int \sin\sqrt{2x+1}\,\mathrm{d}x$ (b) (7%) $\int \tan^{-1}\left(\dfrac{2}{x}\right)\mathrm{d}x$

3. (18%) Compute the integrals.

   (a) (9%) $\int \dfrac{1}{x^2\sqrt{1+x^2}}\,\mathrm{d}x$ (b) (9%) $\int \dfrac{x^2+4x-1}{x^4-1}\,\mathrm{d}x$

4. (18%) For a constant $a > 0$, Ω_a is the region bounded by $y = \dfrac{x}{x+1}$, $x = a$ and the x-axis. Σ_a is the region bounded by $y = \dfrac{x}{x+1}$, $y = \dfrac{a}{a+1}$ and the y-axis. (Ω_a and Σ_a together make a rectangle as shown in the Figure.) Rotate Ω_a about the x-axis and let the resulting volume be $U(a)$. Rotate Σ_a about the y-axis and let the resulting volume be $V(a)$.



   (a) (8%) Find $U(a)$.

   (b) (8%) Find $V(a)$.

   (c) (2%) Find $U(a) - V(a)$.

5. (12%) Find the following limits.

   (a) (6%) $\lim_{x\to 0} \dfrac{\cos x^2 - 1}{\sin x^4}$ (b) (6%) $\lim_{x\to 1}\left(\dfrac{2}{x^2-1} - \dfrac{1}{\ln x}\right)$

6. (a) (4%) Write down the Taylor series of $\dfrac{-1}{\sqrt{1-x^2}}$ at $x = 0$.

   (b) (6%) Find the Taylor series of $\cos^{-1} x$ at $x = 0$.

   (c) (4%) Write down the 10th-degree Taylor polynomial, $P_{10}(x)$, of $\cos^{-1}(x^2)$ at $x = 0$.

   (Note that you need to compute the exact values of coefficients for part (c). Do not express coefficients in terms of C_n^α only.)

7. (14%) Suppose that $f(x) = \dfrac{1}{2}x^2 + \dfrac{1}{3\cdot 2}x^3 + \cdots + \dfrac{1}{n(n-1)}x^n + \cdots$, for $|x| < 1$.

   (a) (2%) Find $f^{(10)}(0)$

   (b) (6%) Find the Taylor series of $f'(x)$ and $f''(x)$ at $x = 0$. Recognize them as elementary functions.

   (c) (6%) Express $f(x)$ as an elementary function.