1. (12%) Let $g(t) = f(x(t), y(t))$, where $\begin{cases} x(t) = a + th \\ y(t) = b + tk \end{cases}$ $a, b, h, k \in \mathbb{R}$. Find $g''(0)$.

2. (14%) Let $f(x, y, z) = z\tan^{-1}(xy)$.
   (a) (6%) Find $\nabla f(2, \frac{1}{2}, 4)$.
   (b) (4%) Suppose that at point $(2, \frac{1}{2}, 4)$ the directional derivative of f in the direction of vector $(a, 1, 0)$ is 0. Find the value of a.
   (c) (4%) Find the tangent plane of the surface $f(x, y, z) = \pi$ at point $(2, \frac{1}{2}, 4)$.

3. (18%) Suppose that $(-2, 1)$ is a critical point of $f(x, y) = x^2 + 4x + y^3 - 3ay$.
   (a) (2%) Find the value of a.
   (b) (6%) Find and classify all critical points of $f(x, y)$.
   (c) (10%) Find the maximum and minimum value of $f(x, y)$ on the rectangle $R = \{(x, y) | -3 \le x \le 0, -2 \le y \le 2\}$.

4. (16%) On the ellipse $x^2 + 4y^2 = 1$, find the maximum and minimum value of $-x^2 + 4xy + 2y^2$.

5. (14%) Compute the integrals.
   (a) (6%) $\int_0^1 \int_{\sqrt[3]{y}}^1 \frac{1}{x(1+x^3)} dx dy$.
   (b) (8%) $\iint_\Omega y^2 e^{xy^2+x} dA$, where Ω is the region bounded by $x = 0$, $y = 0$, $y = \sqrt{3}$ and $x = \frac{1}{1+y^2}$.

6. (14%) Evaluate the double integral $\iint_\Omega x dA$, where Ω is given in terms of polar coordinates by $0 \le \theta \le \frac{\pi}{2}$ and $0 \le r \le \sin 2\theta$.

7. (12%) Evaluate the double integral $\iint_\Omega e^{xy} dA$, where Ω is the region enclosed by $y = 1$, $y = 3$, $xy = 1$ and $xy = 4$.