

1. (14%) Find the following limits.

(a) (7%) $\lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x}$.

(b) (7%) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{3x}$. (Hint: $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$)

2. (14%) Find the first derivative of the following functions.

(a) (7%) $f(x) = \tan(2^x)$.

(b) (7%) $f(x) = (\sin x)^x$.

3. (12%) Given $\tan^{-1} \frac{y}{x} = 2xy - 2y^2 + \frac{\pi}{4}$, find the first derivative and the second derivative, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ at $(1, 1)$.

4. (8%) Estimate $\sqrt[4]{10004}$ by a linear approximation.

5. (8%) Show that $-\ln(1-x) > x + \frac{1}{2}x^2$ for $0 < x < 1$.

6. (14%) As in the picture, consider a trapezoid inscribed in the unit circle such that one base is the diameter.

(a) (4%) Describe the area of the trapezoid as a function of θ . Denote this function by $f(\theta)$.

(b) (7%) Find critical numbers of $f(\theta)$ for $0 < \theta < \frac{\pi}{2}$.

(c) (3%) Find the absolute maximum value of $f(\theta)$ for $0 \leq \theta \leq \frac{\pi}{2}$.

7. (14%) Let $y = f(x) = \sqrt{4x^2 + x}$, for $x \leq -\frac{1}{4}$ or $x \geq 0$. Find slant asymptotes of $y = f(x)$.

8. (16%) Consider $f(x) = 3 \ln(x^2 - 1) - 4x$.

(a) The domain of $f(x)$ is _____.

(b) $f'(x) =$ _____. $f(x)$ is increasing on _____ (intervals).

$f(x)$ is decreasing on _____ (intervals).

(c) $f''(x) =$ _____.

$f(x)$ is concave upward on _____ (intervals, if any).

$f(x)$ is concave downward on _____ (intervals, if any).

(d) At $x =$ _____, $f(x)$ has local maximum value _____. (If there is any local maximum value.)

At $x =$ _____, $f(x)$ has local minimum value _____. (If there is any local minimum value.)

(e) Find vertical asymptotes of $y = f(x)$.

(f) Draw the graph of $y = f(x)$.