1. (12%) Suppose that a continuous function $f(x)$ satisfies the equation $\int_a^{x^2} \frac{f(\sqrt{t})}{t} dt + 2 = x$, for all $x > 0$. Find $f(x)$ for $x > 0$ and the constant $a > 0$.

2. (24%) Evaluate the integrals.

   (a) (8%) $\int_0^1 \sin^{-1} x \, dx$. (b) (8%) $\int \tan^4 x \, dx$. (c) (8%) $\int \frac{1}{x^4 - 1} dx$.

3. (12%) Evaluate the improper integral $\int_0^\infty e^{-x} \sin x \, dx$.

4. (8%) Find the arc length of the curve $y = \frac{e^{2x} + e^{-2x}}{4}$ from $x = 0$ to $x = 1$.

5. (12%) Compute the volume of the solid obtained by rotating the disc, $x^2 + (y-2)^2 \le 1$, about the x-axis.

6. (16%)

   (a) (8%) Write down the Taylor expansion of $(1 - x^2)^{-\frac{1}{2}}$ at $x = 0$.

   (b) (8%) Derive from a) the Taylor expansion of $\sin^{-1} x$ at $x = 0$ and write down the first three nonzero terms.

7. (16%) Compute the following limits.

   (a) (8%) $\lim_{x \to 0} \frac{\sin(x^2)}{e^x - 1 - x}$. (b) (8%) $\lim_{x \to \infty} (1 + \sqrt{x})^{\frac{1}{\ln x}}$.