

1. (13%) Evaluate  $\iint_{\Omega} (x - y)^{20} dA$ , where  $\Omega$  is the parallelogram enclosed by  $2x + y = 0$ ,  $2x + y = 1$ ,  $x + 2y = 0$  and  $x + 2y = 1$ .
2. (12%) Evaluate 
$$\iint_{\Omega} e^{-(x^2+y^2)} dA, \quad \Omega = \{(x, y) : 1 \leq x^2 + y^2 \leq 2, y \geq 0, y \geq x \geq -y\}.$$
3. (13%) Find the points on the curve  $10x^2 + 12xy + 5y^2 = 14$  that are closest to and farthest from  $(0, 0)$ .
4. (12%) Given a function  $f(x, y)$  in two variables  $x, y$ , consider the function  $g(t) = f(1 + 2t, 3 + 4t)$ . Express  $g'(0)$  and  $g''(0)$  in terms of  $\frac{\partial f}{\partial x}(1, 3)$ ,  $\frac{\partial f}{\partial y}(1, 3)$ ,  $\frac{\partial^2 f}{\partial x^2}(1, 3)$ ,  $\frac{\partial^2 f}{\partial x \partial y}(1, 3)$ ,  $\frac{\partial^2 f}{\partial y^2}(1, 3)$ .
5. (12%) Find an equation for the tangent plan to the surface  $xe^{yz} + \ln(y^3z^2) = \tan^{-1}\left(\frac{x}{z}\right)$  at the point  $(0, 1, 1)$ .
6. (13%) Let  $f(x, y) = \left(\frac{x^2}{4} + y^2 - 1\right)^2 + x^2y^2$ .
  - (a) Find  $\nabla f(2, 1)$ .
  - (b) Find  $\frac{\partial f}{\partial \vec{u}}(2, 1)$ , where  $\vec{u}$  is the unit vector in the direction of the vector  $(3, 1)$ .
  - (c) Find the unit vector  $\vec{u}$  such that  $\frac{\partial f}{\partial \vec{u}}(2, 1)$  attains its maximum. Also find this maximum.
  - (d) Find the equation of the tangent line to the level curve of  $f(x, y)$  through the point  $(2, 1)$  at the point  $(2, 1)$ .
7. (13%) Find the critical points of  $f(x, y) = e^{y^2-x^2}(y^2 + x^2)$  and determine it is local maximum, local minimum or saddle points.
8. (12%) Find  $\iint_{\Omega} \frac{\sin x}{x} dA$  where  $\Omega$  is the triangle in the  $xy$ -plane bounded by the  $x$ -axis, the line  $y = x$ , and the line  $x = 1$ .