1. (10%)
   (a) Evaluate $\lim_{x\to 1} \dfrac{(x+1)(3^{x^2+1}-9)}{x^2-1}$.
   (b) Evaluate $\lim_{x\to 2} \dfrac{\tan^3(\pi(x-2))}{(x-2)^3 x}$.

2. (10%)
   (a) State the Mean Value Theorem.
   (b) Show that $\tan^{-1} y - \tan^{-1} x \le y - x$ for $y \ge x \ge 0$.

3. (10%)
   (a) Let $h(x) = x^x$. Find $h'(2)$.
   (b) Let $f(x) = 2^x$. Find $f^{(5)}(x)$.

4. (10%) Evaluate $\dfrac{d}{dx}(\cos\tan^{-1} x)$.

5. (10%) Use linear approximation to estimate $\sqrt{1+\sin(0.003)}$.

6. (14%) By implicit differentiation of $y^3 + xy + x^2 y - x^3 = -1$, find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ at the point $x = 1$, $y = 0$.

7. (23%) Let $y = f(x) = \dfrac{9(x^2-3)}{x^3}$. Find the following
   (a) the intervals on which $y = f(x)$ increases _____ (3%)

   the intervals on which $y = f(x)$ decreases _____ (3%)
   (b) the intervals on which $y = f(x)$ is concave up _____ (3%)

   the intervals on which $y = f(x)$ is concave down _____ (3%)
   (c) the local maximum(if exists) of $y = f(x)$: _____ (coordinates) (2%)

   the local minimum(if exists) of $y = f(x)$: _____ (coordinates) (2%)
   (d) all asymptotes of $y = f(x)$ _____ (4%)
   (e) Sketch the graph of $y = f(x)$. (3%)

8. (13%) We make a drinking cup from a circular piece of paper of radius 3 by first cutting out a sector AOB, where O is the center, and then joining the edges $\overline{OA}$ and $\overline{OB}$. Find the maximum capacity of such a cup. The volume of a cone is $\dfrac{\pi r^2 h}{3}$, where h is its height and r is the radius of its base.