1. (15%)
   (a) (5%) State the fundamental theorem of calculus.
   (b) (10%) A function $f(x)$ satisfies

$$\int_0^x f(t)dt = \int_x^1 t^2 f(t)dt + \frac{x^{16}}{8} + \frac{x^{18}}{9} + C \qquad \text{for all } x,$$

   where C is a constant. Find the function $f(x)$ and the constant C.

2. (7%) Evaluate $\lim_{x\to\infty} \dfrac{\ln(1+x)}{\ln(1+x^2)}$.

3. (7%) Evaluate $\lim_{x\to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{\sin^2 x}\right)$.

4. (14%)
   (a) (7%) Derive the Taylor expansion of $\tan^{-1} x$ centered at $x = 0$. (The expression of the n-th term for general $n \geq 0$ is required.)
   (b) (7%) Find $\lim_{x\to 0} \dfrac{3\tan^{-1} x - 3x + x^3}{3x^5}$.

5. (14%) Evaluate the following two indefinite integrals.
   (a) (7%) $\int x \sin^{-1} x dx$.
   (b) (7%) $\int \dfrac{\ln x}{x \ln x + x} dx$.

6. (6%) Find the n-th term of the Taylor expansions of $\sin(x^2)$ centered at $x = 0$ for general $n \geq 0$. (*You may use the Taylor expansions of* $\cos x$ *and* $\sin x$ *without deriving them.*)

7. (7%) Find the n-th term of the Taylor expansion of $\sin^2 x$ centered at $x = 0$ for general $n \geq 0$. (*You may use the Taylor expansions of* $\cos x$ *and* $\sin x$ *without deriving them.*)

8. (10%) Find the arc length of the curve $y = \ln\sec x$ for $0 \leq x \leq \dfrac{\pi}{4}$. (You can use any integral formulas you know without deriving them.)

9. (20%) Let Ω be the region bounded by $y = \dfrac{1}{x(3-x)}$, $x = 1$, $x = 2$ and x-axis.
   (a) (7%) Find the area of Ω.
   (b) (7%) Find the volume of the solid obtained by rotating Ω about the x-axis.
   (c) (6%) Find the volume of the solid obtained by rotating Ω about the y-axis.