1. (10%) Solve the differential equation $y' = y(y^2 - 1)$, where y is not a constant function.

2. (10%) Find the particular solution of $x\dfrac{dy}{dx} = 2y + x^3 \ln x$, $x > 0$, satisfying $y(1) = -1$.

3. (15%) Assume the range I_X of a random variable X is $\{-1, 0, 1\}$. Let $\mathrm{E}(X) = 0$ and $\mathrm{Var}(X) = \dfrac{2}{3}$. Find
   (a) (4%+4%) $P(X = 1)$ and $P(X = 0)$
   (b) (7%) $\mathrm{Var}(X^2)$.

4. (10%) If $X_i \sim X$ for all $i = 1, 2, \cdots, 10$ and $\{X_1, X_2, \cdots, X_{10}\}$ is a set of independent random variable. Assume $\mathrm{E}(X) = 1$ and $\mathrm{Var}(X) = 2$. Find
   (a) (5%) $\mathrm{E}(5X_1 \cdot X_2 \cdot \cdots \cdot X_{10})$
   (b) (5%) $\mathrm{Var}\left(\dfrac{X_1 + X_2 + X_{10}}{10}\right)$

5. (10%) Compute the integral: $\displaystyle\int_{-\infty}^{\infty} (2x+1)e^{-x^2+6x}\,dx$.(You can use the result $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$.)

6. (15%) Let X, Y be two independent random variables with density function $f_X(t) = f_Y(t) = \begin{cases} te^{-t}, & t > 0 \\ 0, & t \le 0. \end{cases}$.
   (a) (8%) Let $Z = X + Y$. Find $f_Z(t)$.
   (b) (7%) Let $W = X^2$. Find $f_W(t)$.

7. (15%) (5%+5%+5%) Let X be a random variable with density function $f_X(t) = 3e^{-3t}$. Find (a) $P(1 \le X \le 2)$, (b) $\mathrm{E}(X)$ and (c) $\mathrm{Var}(X)$.

8. (15%) Calls arrive to a cell in a certain wireless communication system according to a Poisson process with arrival rate 120 calls per hour. Find the following probabilities:
   (a) (7%) The first call arrives after time $t = 3.5$.
   (b) (8%) Two or fewer calls arrive in the first 3.5 minutes.