

1. Let $f(x) = x\sqrt{x^2 + 3}$.
 - (a) (5%) Show that $f(x)$ is a one-to-one function.
 - (b) (5%) Let $g(x)$ be the inverse function of $f(x)$. Find $g'(-2)$.
2. (a) (6%) Set $a, b \in \mathbb{R}$. If $\lim_{x \rightarrow 0} \frac{\sqrt{ax + b} - 2}{x} = 2$, then $a, b = ?$
 - (b) (6%) Find $\lim_{n \rightarrow \infty} \left(\frac{n+2}{n-2}\right)^{2n}$.
3. (a) (5%) Let $y = \cot(\cos^2(3x))$. Find $\frac{dy}{dx}$.
 - (b) (5%) Let $y = x^{\ln x}$, $x > 0$. Find $\frac{dy}{dx}$.
 - (c) (5%) Find the 52th derivative of $\sin 3x$.
4. (10%) Prove the inequality $|\tan a - \tan b| \geq |a - b|$ for $a, b \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $a \neq b$.
5. (10%) Use linear approximation to estimate the value of $e^{\sin^{-1}(-0.0002)}$.
6. (10%) Find an equation of the tangent line to the curve $x \cos^2 y = \sin y$ at the point $(0, \pi)$.
7. (23%) Let $y = f(x) = \frac{(x+1)^3}{x^2 + 2x}$. Find the following
 - (a) the intervals on which $y = f(x)$ increases _____ (3%)
 the intervals on which $y = f(x)$ decreases _____ (3%)
 - (b) the intervals on which $y = f(x)$ is concave up _____ (3%)
 the intervals on which $y = f(x)$ is concave down _____ (3%)
 - (c) the local maximum(if exists) of $y = f(x)$: _____ (coordinates) (2%)
 the local minimum(if exists) of $y = f(x)$: _____ (coordinates) (2%)
 - (d) all asymptotes of $y = f(x)$ _____ (4%)
 - (e) Sketch the graph of $y = f(x)$. (3%)
8. (10%) A truck gets $500/x$ kilometers per liter when driven at a constant speed of x kph (between 60 and 120 kph). If the price of fuel is \$20/liter and the driver is paid \$400/hour, at what speed between 60 and 120 kph is it most economical(minimal sum of the fuel cost and driver's pay) to drive for 400 kilometers?